WAVE AND STABILITY IN FLUIDS

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PREFACE

The book is based on the courses given by the first author (DYH) over the years at Brown University, Wuhan University and the Hong Kong University of Science and Technology. It is designed for students in applied mathematics and engineering, who have already been exposed to basic fluid dynamics. The main purpose of the book is to provide relatively easy access to the understanding of the essence of the mechanisms involved in waves and stabilities in fluid. The contents of the book is inevitably influenced by the interests and biases of the authors.

Except for Sections 4.1, 4.2 and 4.3, the writing of the book is essentially carried out by the first author. He has to bear almost all the responsibility of the shortcomings of the book.

The book is largely self-contained. References are given only for supplementation of missing details which cannot be found readily in familiar books such as that listed at the end of the book.

We would like to acknowledge the contributions of graphs in Chapter 6 to our colleague, Dr. Xiao-Ping Wang, and to Mr. Chow Chak On, a second year undergraduate student at HKUST.

Our special thanks to Miss Grace Yeung who typed and produced this camera-ready manuscript, and ingeniously helped us to solve many problem relating to the production of the book.

D.Y. HSIEH
S.P. HO

Hong Kong
June 1994

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