WAVE AND STABILITY IN FLUIDS

This page is intentionally left blank

WAVE AND STABILITY IN FLUIDS

D. Y. Hsieh

The Hong Kong University of Science and Technology

S. P. Ho

Hong Kong Polytechnic

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication Data

Hsieh, D. Y. (Din-Yu), 1933–

Wave and stability in fluids / D.Y. Hsieh, S.P. Ho.

p. cm.

Includes bibliographical references and index.

ISBN 9810218702

1. Fluid dynamics. 2. Waves. 3. Stability. I. Ho, S. P.

II. Title.

QC151.H75 1994

532'.059--dc20

94-30338

CIP

Copyright © 1994 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 27 Congress Street, Salem, MA 01970, USA.

Printed in Singapore by Uto-Print

PREFACE

The book is based on the courses given by the first author (DYH) over the years at Brown University, Wuhan University and the Hong Kong University of Science and Technology. It is designed for students in applied mathematics and engineering, who have already been exposed to basic fluid dynamics. The main purpose of the book is to provide relatively easy access to the understanding of the essence of the mechanisms involved in waves and stabilities in fluid. The contents of the book is inevitably influenced by the interests and biases of the authors.

Except for Sections 4.1, 4.2 and 4.3, the writing of the book is essentially carried out by the first author. He has to bear almost all the responsibility of the shortcomings of the book.

The book is largely self-contained. References are given only for supplementation of missing details which cannot be found readily in familiar books such as that listed at the end of the book.

We would like to acknowledge the contributions of graphs in Chapter 6 to our colleague, Dr. Xiao-Ping Wang, and to Mr. Chow Chak On, a second year undergraduate student at HKUST.

Our special thanks to Miss Grace Yeung who typed and produced this camera-ready manuscript, and ingeniously helped us to solve many problem relating to the production of the book.

D.Y. HSIEH
S.P. HO

*Hong Kong
June 1994*

This page is intentionally left blank

TABLE OF CONTENTS

Preface	v
1. Preliminaries on Waves	1
1.1 Governing Equations of the Fluid Motion, 2 <i>Eulerian and Lagrangian Coordinates</i> , 6	
1.2 The Wave Equation, 8	
1.3 Simple Solutions : Sinusoidal Waves, 10	
1.4 Fundamental Solutions of Wave Equation, 14	
2. Sound Waves	21
2.1 Waves in Compressible Fluids, 21	
2.2 Multi-pole Radiation, 23	
2.3 Generation of Sound, 27	
2.4 Scattering of Sound, 34 <i>Scattering by a Spherical Bubble in a Liquid</i> , 39 <i>Scattering by a Small Body</i> , 42	
2.5 Nonlinear Acoustics, 45 <i>Quadrapole Radiation</i> , 45 <i>Scattering of Sound by Sound</i> , 48 <i>Acoustic Streaming</i> , 53	
2.6 Simple Wave and Shock Wave, 56	
2.7 Structure of Shock - Burgers Equation, 69	

Wave and Stability in Fluids Downloaded from www.worldscientific.com
by 18.209.158.208 on 04/26/23. Re-use and distribution is strictly not permitted, except for Open Access articles.

4.	Waves in Nonhomogeneous Media	170
4.1	Internal Waves of Layered Fluid Media, 170 <i>Upper Fluid Surface Free</i> , 176	
4.2	Waves in Continuously Stratified Fluids, 179 <i>The Case that $\rho_0 = \rho^{(0)}e^{-\beta z}$</i> , 183 <i>Rigid Surface on Top</i> , 184 <i>Free Surface on Top</i> , 185	
4.3	Waves in Compressible Stratified Fluids, 188	
4.4	Dynamic Equations of Bubbly Liquids, 199 <i>A Simple Model of Bubbly Liquids</i> , 201	
4.5	Waves in Bubbly Liquids, 202 <i>Sound Waves</i> , 202 <i>Dead Zone and Negative Damping</i> , 209 <i>Waves in Locked Bubbly Liquids</i> , 211	
5.	Stability	216
5.1	Interfacial Conditions, 217	
5.2	The Classical Rayleigh-Taylor Stability, 223	
5.3	Rayleigh-Taylor Stability with Mass and Heat Transfer Across the Interface, 229 <i>A Simplified Version</i> , 236 <i>Nonlinear Stability Based on Simplified Version</i> , 239	
5.4	Stability of Spherical Bubble in Motion, 244 <i>Stability of Expanding and Collapsing Bubbles</i> , 252	

- 5.5 The Classical Kelvin-Helmholtz Stability, 254
- 5.6 Variational Method and Kelvin-Helmholtz Stability, 262
 - Nonlinear Analysis*, 268
 - Fully Nonlinear One-Mode Analysis*, 271
- 5.7 Kelvin-Helmholtz Stability for Compressible Fluids, 272
 - The Case of Semi-infinite Fluid Layers*, 277
 - Relationship with Two Phase Flows*, 281
- 5.8 The Faraday Problem, 285
 - Linear Stability*, 288
 - Nonlinear Theory*, 292
 - Stability of Oscillating Bubbles*, 294
- 5.9 The Rayleigh-Benard Problem, 296
 - Principle of Exchange of Stabilities*, 304
 - Critical Rayleigh Number*, 307
 - The Cell Patterns*, 310
 - Non-uniform Heating*, 312
- 5.10 The Taylor-Couette Problem, 313
 - Rayleigh's Criterion*, 316
 - Linear Stability Analysis*, 319
 - Narrow Gap*, 323
- 5.11 Stability of Parallel Flows, 327
 - Squire Theorem*, 330
 - Orr-Sommerfeld Equation*, 331
 - Sufficient Conditions for Stability*, 332

	<i>Parallel Flow of Inviscid Fluids</i> , 334	
	<i>Asymptotic Analysis and Comparison</i>	
	<i>Equations</i> , 337	
5.12	Stability of Flow Down an Inclined Plane, 344	
	<i>Small k Expansion</i> , 349	
	<i>Hydraulic Approximation</i> , 353	
6.	Chaos	360
6.1	The Lorenz Equations, 361	
6.2	The Logistic Map, 375	
6.3	Characterization of Chaos, 384	
	<i>Lyapunov Exponents</i> , 384	
	<i>Dimensions</i> , 387	
	<i>Dynamical Information in Experimental Data</i> , 390	
6.4	Almost Ill-posed Problems and Chaos, 391	
	<i>Examples of Underlying Ellipticity</i> , 394	
	<i>Model Partial Differential Equations with Underlying</i>	
	<i>Ellipticity</i> , 398	
Appendix 1	<i>Some Properties of Spherical Bessel Functions</i>	402
Appendix 2	<i>On Surface Tension</i>	404
	Books for Concurrent and Further Reading	407
	References	409
	Index	411