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TOWARDS A MATHEMATICAL THEORY OF COMPLEX BIOLOGICAL SYSTEMS

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TOWARDS A MATHEMATICAL THEORY OF COMPLEX BIOLOGICAL SYSTEMS

C Bianca • N Bellomo

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Preface

This monograph focuses on the ambitious aim of developing a mathematical theory of complex biological systems with special attention on the phenomena of ageing, degeneration, and repair of biological tissues under individual self-repair actions that may also take advantage of medical actions and therapies. The immune system plays an essential role in the competition against pathologies. Of course, the authors do not naively claim that such objective is effectively achieved. Simply some perspective ideas, and some preliminary steps, are brought to the attention of applied mathematicians.

The approach to the mathematical modeling of biological systems needs to tackle the additional difficulty generated by the peculiarity features of living matter. For instance the lack of invariance principles which are typical of systems pertaining to inert matter, the ability to express a strategy for individual well-being, heterogeneous behaviors, competition, proliferative and/or destructive actions, learning ability, evolution, and many others.

The presentation is organized in three parts with ten chapters, which have the aim of addressing applied mathematicians to research activity in the field of modeling complex biological systems viewed as living, and hence biological systems:

Part I is devoted to a phenomenological description of the aforementioned biological processes with the aim of capturing the characteristics that play a crucial role in the modeling process. A new systems biology approach is subsequently proposed, where the overall system is decomposed into subsystems on the basis of the biological functions that are expressed by each subsystem.

Part II presents the mathematical tools that are designed to achieve the aims of the monograph. These tools define a class of integro-differential equations that offer an appropriate underlying mathematical structure that can be used to derive models at the molecular and cellular scale. Moreover, mathematical methods to

link the dynamics from the molecular to the cellular scale, and from this to that of tissues are studied in view of applications.

Part III deals with applications and perspectives. First, the modeling of two case studies is presented, namely the modeling of malignant keloid formation, and the derivation of chemotaxis models at the macroscopic scale from the underlying description at the cellular scale. Subsequently a critical analysis is proposed to understand how much work is still needed to achieve a mathematical theory of biological systems, and to identify the guidelines necessary to pursue this objective.

In more details, the sequential steps of the mathematical approach are as follows:

- i) *Modeling biological systems* as a large, heterogeneous, multiscale complex system;
- ii) Integration of the concept of *functional subsystems* to reduce the complexity of large living systems within the approach of systems biology;
- iii) Development of the *mathematical kinetic and stochastic game theory for active particles*;
- iv) Modeling interactions at the molecular and cellular scale and application of the theoretical approach to some case studies;
- v) Development of a *multiscale analysis and modeling from genes to tissues*.

The final aim of this monograph is to provide a new conceptual background for applied mathematicians involved in the challenging research field of mathematics of living systems. The authors trust that the validity of the methodological will not be limited to the specific cases treated in the monograph, but it will provide the background for a large variety of problems generated by the interdisciplinary approach of mathematics and biology.

The authors are indebted to the Partners of the Project RESOLVE of the European Union. Their contribution to understand complex biological phenomena and to their modeling by an interdisciplinary mathematical-biological approach has been a precious gift.

Torino,
September 2010

Carlo Bianca
Nicola Bellomo

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