INCOMPLETE BLOCK DESIGNS

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Aloke Dey

Indian Statistical Institute, New Delhi, India

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Preface

Block designs have applications in almost all areas of human investigation including agriculture, biology, engineering, medicine, physical and chemical sciences and industrial experimentation. The most primitive of the block designs is the randomized (complete) block design. However, in many practical situations, adoption of a complete block design is not appropriate and in some cases, not at all feasible. This fact prompted the development of various kinds of incomplete block designs, which in turn have been used extensively for experiments in a variety of fields. Moreover, these designs opened up many challenging problems in combinatorial mathematics. In view of the importance of block designs both from a theoretical and practical perspective, the author published a book *Theory of Block Designs* in 1986, which was well received in academic circles. However, the book went out of print around 1992. The author initially toyed with the idea of bringing out a second edition of the book, incorporating only minor additions/changes. While attempting to do so, however, it was realized that during the intervening period, the subject has grown considerably and the emphasis on certain topics has shifted. The author therefore decided to write the present book which, while retaining some of the flavor of the earlier book, is substantially different from it in both coverage and presentation.

The literature on incomplete block designs is vast and it is near impossible to cover each and every development in incomplete block designs in a single book of reasonable length. In this book, an attempt has been made to cover all the developments in this area which in the author's perception are the major ones. Since the classical incomplete block designs like the balanced incomplete block and partially balanced incomplete block designs are still found useful in several applications and newer applications of these, e.g., in visual cryptography, have been found, such designs have been covered at some length. Some of the more recent developments in incomplete block designs for special types of experiments, like biological assays and diallel crosses have also been discussed. Important results on the optimality aspects of various incom-

plete block designs are also reviewed.

The book is organized into six chapters followed by an appendix. A brief description of the chapter contents appears in Chapter 1. The appendix covers some essentials on linear algebra, linear statistical models, finite fields and finite geometries. There are a large number of exercises at the end of Chapters 2–6 and there is a fairly exhaustive bibliography.

Throughout, results in matrix theory are used extensively and thus, a background in basic linear algebra and theory of matrices will be helpful in reading the book. Familiarity with the general area experimental designs and linear statistical models at an advanced undergraduate level is also assumed.

The book can be used in a variety of ways. The material in Chapter 2, the first four sections of Chapter 3 and Sections 4.1–4.4 and 4.6 of Chapter 4 can provide a solid foundation of the theory of incomplete block designs for a master's level course. The material in Sections 3.5–3.7, 4.7–4.11 and that in Chapters 5 and 6 may be used as a basis of a more advanced course. While the book is addressed to an audience whose primary interest is in the theory and applications of statistical design of experiments, portions of the book can also be used for a course in combinatorial designs for mathematicians. The material in Chapters 5 and 6 may be found useful for research students and consulting statisticians.

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Aloke Dey

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