THERMOMECHANICS OF PHASE TRANSITIONS IN CLASSICAL FIELD THEORY

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THERMOMECHANICS OF PHASE TRANSITIONS IN CLASSICAL FIELD THEORY

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PREFACE

The phase transitions in a substance $\mathcal{C}$ are due to certain weak interactions among the molecules of $\mathcal{C}$ which might correctly be described by Quantum Mechanics. However, if we abstain from understanding every minute detail of behaviours in phase transitions, we can limit ourselves to describe these phenomena by resorting to much simpler and tractable schemes. In other words, we use a macroscopic description which is unable to explain either why a state change takes place or which modifications this produces at a microscopic level.

The first approach starts from the experimental observation that the different phases are separated by very narrow layers across which the fields associated to $\mathcal{C}$ vary continuously but sharply. Owing to the high values assumed by the gradients of the fields in these layers, the ordinary constitutive equations are supposed to depend weakly on the higher order gradients of the aforesaid fields. This, in turn, implies that the highest order derivatives in the local equations of balance are multiplied by small coefficients. As is well-known, this property leads just to boundary layers whose localization and forms depend on the equations as well as on the region occupied by $\mathcal{C}$ (see for instance [50,51]).

Another approach to attain a macroscopic description of phase transitions is obtained by resorting to the models of continua with an interface. The basic idea of this approach is that we can substitute the narrow layers between the phases with surfaces of discontinuity for the volume fields provided that we associate to these surfaces some physical attributes which evoke the complex structure of the fields in the layers they substitute. This essay is just dedicated to this approach without any attempt to compare it with the alternative approach of boundary layers (see for instance [52] for a comparison).

After some kinematical preliminaries (chapters 1 and 2), the detailed analysis of the balance equations for a system with an interface as well as of the average procedures which allow to define the physical attributes to associate to the interface are discussed in chapter 3. The remaining chapters are dedicated to the applications of the general scheme to the equilibrium and thermodynamical evolution of phase transitions in liquids, solids, mixtures, crystals and ferroelectric materials. For the phase transitions in these last two materials a suitable use of the nonlocal theories of Continuum Mechanics is suggested.

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