Adaptive High-Order Methods in Computational Fluid Dynamics

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To My Family

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Preface

This book contains invited chapters written by leading international experts on adaptive high-order methods in computational fluid dynamics (CFD). It covers several widely used, and still intensively researched methods, including the discontinuous Galerkin (DG), residual distribution, differential quadrature, k-exact finite volume, spectral volume/spectral difference, $P_N P_M$, and correction procedure via reconstruction methods. The reasons for including such a wide coverage of methods are to: (1) provide a single source of reference, (2) present a snapshot of the state-of-the-art, and (3) facilitate the observation of similarities and differences as well as pros and cons of these methods.

In the present context, adaptive high-order methods refer to numerical methods that are capable of handling unstructured adaptive meshes with accuracy higher than second-order. These methods are compact, scalable, capable of handling both complex physics and geometry, and suitable for modern parallel supercomputers and graphics processing units (GPUs). They are widely considered the next major breakthrough in CFD, and have already found applications in computational aeroacoustics, computational electromagnetics, vortex dominated flows, and large eddy simulation and direct numerical simulation of turbulent flows.

A concerted effort was made to minimize overlaps among the chapters. For example, the first 7 chapters describe different aspects of the DG methods, while the last 8 chapters are devoted to other high-order methods. Main topics covered include innovative formulations, analyses, efficient solution and time marching algorithms, parallel implementation, turbulence modeling, discontinuity-capturing techniques, error estimates, hp-adaptations, and dynamic mesh techniques, etc.

The book requires a graduate student level of understanding. It should serve as an excellent source of information for CFD developers, educators, researchers, users, and students who are interested in the state-of-the-art and the remaining challenges in adaptive high-order methods.

I am grateful to Dr. Chang Shu, a close friend and the Co-Editor-in-Chief of the book series, *Advances in Computational Fluid Dynamics* in World Scientific, for suggesting the book. Heartfelt thanks are due to all the contributors of this volume. Needless to say, the book would not exist without their hard work. Finally, I'd like to thank Ying Zhou for producing the color cover graphic, and Varun Vikas for help with Latex.

Z.J. Wang
Ames, Iowa
June 30, 2011

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