Analysis and Control Using the
Lambert W Function

TIME-DELAY SYSTEMS

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To our families, with love

Preface

This book is a collection of recent research work on the development of an analytical approach for solutions of delay differential equations via the Lambert W function. It, also, includes methods for analysis and control based on the solutions, and their applications to mechanical and biological systems.

Delay differential equations represent systems that include inherent time-delays in the system or a deliberate introduction of time-delays for control purposes. Such time-delays, common in systems in engineering and science, can cause some significant problems (e.g., instability and inaccuracy) and, thus, limit and degrade the achievable performance of controlled systems. However, due to their innate complexity including infinite-dimensionality, it is not feasible to analyze such systems with classical methods developed for ordinary differential equations (ODEs).

The research presented in this book uses the Lambert W function to obtain free and forced closed-form solutions to such systems. Hence, it provides a more analytical and effective way to treat time-delay systems. The advantage of this approach stems from the fact that the closed-form solution is an infinite series expressed in terms of the parameters of the system. Thus, one can explicitly determine how the parameters are involved in the solution. Furthermore, one can determine how each system parameter affects the eigenvalues of the system. Also, each eigenvalue in the infinite eigenspectrum is associated individually with a branch of the Lambert W function.

The Lambert W function-based approach for the analytical solution to systems of delay differential equations (DDEs) had previously been developed for homogeneous first-order scalar and some special cases of systems of delay differential equations using the Lambert W function as introduced

in Chapter 1. In Chapter 2, the analytical solution is extended to the more general case where the coefficient matrices do not necessarily commute, and to the nonhomogeneous case. The solution is in the form of an infinite series of modes written in terms of the matrix Lambert W function. The derived solution is used to investigate the stability of time-delay systems via dominant eigenvalues in terms of the Lambert W function. It is also applied to the regenerative machine tool chatter problem of a manufacturing process in Chapter 3. Based on the solution form in terms of the matrix Lambert W function, algebraic conditions and Gramians for controllability and observability of DDEs are derived in a manner analogous to the well-known controllability and observability results for ODEs in Chapter 4. In Chapter 5, the problem of feedback controller design via eigenvalue assignment for linear time-invariant time-delay systems is considered. The method for eigenvalue assignment is extended to design robust controllers for time-delay systems with uncertainty and to improve transient response in Chapter 6. For systems where all state variables cannot be measured directly, a new approach for observer-based feedback control is developed and applied to diesel engine control in Chapter 7. In Chapter 8, the approach using the Lambert W function is applied to analyze a HIV pathogenesis dynamic model with an intracellular delay.

The authors hope that this book will be of interest to graduate students and researchers in engineering and mathematics who have special interest in studying the properties, and in designing controllers, for time-delay systems.

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S. Yi

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