Stochastic Filtering with Applications in Finance

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To Rajiv and Mitra

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Preface

Stochastic filtering has been in use in engineering and in particular in the area of signal processing for a long time. Its usefulness in financial economics has been recognized for some time. The objective of a filtering application is to determine the best (in some sense) estimate for the states that drive a stochastic dynamical system. The measurable observations that the researchers have are not these states but some linear functions of these states. As an example, we can think of observations on futures prices on some asset, but we do not know anything about the stochastic convenience yield that may be contributing to the futures price formation. Filtering is an iterative process that enables us to estimate the states of the system as well as any hyper-parameters of the models. In the simplest case when there are no gaps in the measurable observations and all observations are made on equal time step, the solution to the problem is the well-known Kalman-Bucy filter. This basic structure, also referred to as state space model, has been found to be a valuable tool within Central Banks as well as financial institutions to deal with difficult inference problems as well for forecasting.

Complications arise when the functions relating to the unobservable states and the measurable observations are not linear. There may be missing observations or the conditional distributions are not necessarily Gaussian or even the time steps between the observations are not equal. To cater to such variations many improvements have been proposed in the literature.

The purpose of this book is to provide a comprehensive account of this very important tool with a view to making it more popular among researchers in finance and economics. It is not intended to give complete

mathematical treatment of different stochastic filtering approaches, but describe them in simple terms and illustrate their application with real historical data for problems normally encountered in these disciplines. Although no prior knowledge in this area is assumed, the readers are expected to have knowledge of probability theory as well as general mathematical aptitude. The book is intended as a reference for graduate students and researchers interested in the field. It is also suitable for use as a text for a graduate level course on stochastic filtering.

The idea of such a book was growing since I started my own PhD research. I came to quantitative finance research after working in the information technology area for several years in technical software development and maintenance in commercial organizations. Besides, my early graduate level education in physics encourages a scientific frame of mind to deal with problems in financial economics. I attempted to synthesize the contents based on my life experiences in a diverse range of activities. I sincerely believe that this book will offer fresh perspectives to the readers in dealing with problems encountered in financial economics. The book is less demanding on mathematical aptitude and at the same time conveys the underlying complexities with the help of examples familiar to the readers in the finance and economics disciplines.

The book aims to be an example-driven elucidation of a complex technical area so that it appeals to a wide variety of readers in the discipline of finance and economics. The algorithmic presentation of the models for the applications will help readers to take up new challenges in their own field of study/research. This will also help them to get a clear understanding of the filtering issues quickly and enhance the models that suit their own perceptions.

The first chapter deciphers the notion of filtering and various ways of estimating models under different assumptions. Once this is understood, the succeeding chapters take up relevant applications in finance and economics and show how filtering framework applies in those situations. The second chapter focuses on the foreign exchange market and extracts embedded risk premium in the forward exchange rates. The third chapter explores the notion of risk premium in the equity market with a difference. It analyzes the price of risk and not just the total quantity of risk.

The fourth chapter revisits an important issue in macroeconomics. This is to do with the notion of inflation uncertainty and its connection with interest rates in the economy. It applies the filtering scheme to extract two forms of uncertainty from the inflation data and then explores its relationship with interest rates with particular reference to countries following inflation targeting policy.

The fifth and sixth chapters move the discussions to the interest rate model proposed by Heath, Jarrow and Morton in 1992. The rich but non-Markovian dynamics of this approach also entail computational and implementation complexities. These chapters describe how to deal with the issues via expansion of states and stochastic filtering. The examples include bond pricing as well as pricing of interest rate futures contracts.

Chapters 7 through 9 concentrate on fixed income securities. The possible drivers of credit spread are analyzed in Chapter 7 in a filtering framework. Chapter 8 takes up the structure of relatively new security in the credit derivatives market, i.e., credit default swaps. It disentangles the component structure of credit default swaps and analyzes the possible explanatory variables for these components. Chapter 9 moves on to explore the model for pricing options on credit default swaps. A relatively new filtering algorithm is exploited here for model calibration.

Chapter 10 explores different methods for estimation of the stochastic variance model applied to short-term interest rates. It also shows how to enhance the time series dynamics of the unobserved variance process.

Chapter 11 moves away from simply diffusion based filtering problems to those where jumps are also involved. This is illustrated with two examples. The first one is to capture the price path dynamics of spot electricity and the second one examines whether the jump risk could be diversified in the market for credit default swaps index and its sub-indices.

It is clear from the breadth of coverage in this book that it could not have been accomplished without the help from others. I would like to thank first my PhD students: Peipei Wang, Nedim Handzic and Damien Lee. I wish them all the success in their future endeavors. Also, I am grateful to a colleague of mine, David Colwell, with whom I had many insightful sessions.

I would also like to express my sincere gratitude to Carl Chiarella, my PhD supervisor at the University of Technology, Sydney. That is when I started learning about the interest rate model due to Heath, Jarrow and Morton and the complexities within the framework.

I am thankful to David Sharp at the World Scientific Press in Singapore for persisting with me to come up with the idea for this book.

Finally, I am grateful to my family for support and patience. My son regularly enquired about the progress of the book while he was studying Engineering at the Imperial College, London.

R. Bhar
Sydney, 2010

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