ELEMENTARY FUNCTIONAL ANALYSIS

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Preface

This text is based on a set of lecture notes for an introductory course in functional analysis which was offered during a summer session at New Mexico State University. Most of the students had only completed a first year course in real analysis and had no background in Lebesgue integration. Although this lack of knowledge of the Lebesgue integral limits the application of the results in abstract functional analysis to problems in concrete function spaces, particularly in partial differential equations, there are still sufficient applications presented in the text to topics in ordinary differential equations, integral equations, harmonic analysis and numerical integration to illustrate the many applications of functional analysis. Only a very basic knowledge of linear algebra and introductory real analysis including a knowledge of metric spaces at the level of Apostol ([Ap]), Bartle ([Ba]), DePree/Swartz ([DeS]) or Rudin ([Ru]) is necessary to follow the developments in the text; at one point in the chapter on spectral theory, Liouville's theorem from complex analysis is used. In the first chapter, we introduce and establish the basic properties of normed spaces and in Chapter 2, we study continuous linear operators between normed spaces and give applications to integral equations. In Chapters 3, 4 and 5, we consider quotient spaces, finite dimensional normed spaces and inner product (Hilbert) spaces. In Chapters 6-12, we establish the 3 basic principles of functional analysis — the Hahn-Banach Theorem, the Uniform Boundedness Principle and the Open Mapping/Closed Graph Theorems — and give applications to Banach limits, normed spaces, Fourier series, summability, weak and weak* convergence, projections and Schauder bases. In Chapter 13, we define the transpose (conjugate, adjoint) of a continuous linear operator between normed spaces and establish some of the relationships between the range and kernel of an operator and its transpose. In Chapter 14, we introduce

the class of compact operators and in Chapter 15, we establish the Fredholm Alternative for compact operators. The remaining chapters of the text are devoted to studying spectral theory for continuous linear operators and giving applications to ordinary differential and integral equations. Chapters 16 and 17 give the definition of the spectrum of a continuous linear operator and examples of spectrums. Chapter 18 contains a description of the spectrum of a compact operator while Chapters 19 and 20 define symmetric operators and establish the spectral theorem for compact, symmetric operators and give applications to integral equations. Chapter 21 gives a spectral representation for operators which have compact inverses and an application of the representation to Sturm-Liouville ordinary differential equations. Chapters 22-25 establish the spectral theorem for bounded self adjoint operators on a Hilbert space and Chapter 26 uses the spectral theorem to define an operational calculus for bounded self adjoint operators; the operational calculus is then employed to establish the existence of the square root of a positive operator and the polar decomposition for a normal operator.

There are a number of excellent texts on functional analysis. There are two things that distinguish this text from other texts in functional analysis. The first is that there are minimal prerequisites, especially there is no requirement of the knowledge of the Lebesgue integral or general topology. There are a number of functional analysis texts which require similar minimal backgrounds such as, Pryce ([Pr]) or Schecter ([Sch]), but these texts do not contain many applications of the abstract results of functional analysis to concrete problems in function spaces whereas we present a large number of applications to ordinary differential and integral equations, Banach limits, Schauder bases, harmonic analysis, summability and numerical integration. On the other hand, there are a large number of functional analysis texts such as Dunford/Schwartz ([DuS]), Rudin ([Ru2]), Taylor/Lay ([TL]), and Yosida ([Y]) which contain many applications of the abstract results of functional analysis to function spaces but which have much more stringent background requirements.

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