METHODS OF WAVE THEORY IN DISPERSIVE MEDIA

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Synopsis

The monograph presents main analytic mathematical methods and general problems in the theory of linear waves in dispersive media and systems, including nonequilibrium ones. To show how the general theory can be applied in practice, a unified description is given of important physical systems that are traditionally studied in the mechanics of continuous media, electrodynamics, plasma physics, electronics, and physical kinetics. An analysis is made of the interaction of waves in coupled systems, the propagation and evolution of localized wave perturbations, and the emission of waves in dispersive media under the action of external sources moving in a prescribed manner. A general theory of instabilities of linear systems is presented in which the criteria for absolute and convective instabilities are formulated and compared, and Green's functions for some nonequilibrium media are calculated. Special attention is paid to problems in the theory of linear electromagnetic waves in plasmas and plasmalike media. The monograph also contains a number of original results of the present-day wave theory that have been published by now in scientific journals only.

The book is aimed at researchers and experts, as well as students and post-graduates, who specialize in such fields as the electrodynamics and mechanics of continuous media, physical electronics, and radiophysics.

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Introduction

The monograph presents mathematical methods of description and general physical results in the theory of linear waves in dispersive media and systems, including nonequilibrium ones. In essence, it gives formulations and solutions of problems for n th order linear partial differential equations and also physical interpretations of the solutions and their practical applications. Since the literature (manuals, monographs, reviews, etc.) on the theory of linear waves is now so extensive that it seems to be exhaustive, the question naturally arises of whether it is expedient to publish books like the one you are reading. We think, however, that books of this kind are still needed. The main reasons are twofold. First, wave theory is traditionally presented in the context of particular physical objects, such as optical waves, radiowaves, plasma waves, waves in fluids, and acoustic waves. But mathematically, wave theory can be constructed and presented irrespective of the physical nature of the wave process. And second, the very important subjects of modern natural sciences are nonequilibrium physical systems, for which wave theory plays a secondary role and is merely part of such original branches of physics as physical kinetics, plasma physics, microwave electronics, to name but a few. Yet, there is clearly a need for an original theory of waves in nonequilibrium media. In our monograph, the general theory of linear waves is presented just as a branch of mathematical physics that describes the dynamics of linear waves in equilibrium and nonequilibrium dispersive media and systems, irrespective of their physical nature. The practical application of the general theory is illustrated by considering fairly simple but important physical systems that are traditionally studied in the mechanics of continuous media, electrodynamics, plasma physics, and electronics. The practical problems are solved by a unified approach presented in the mathematical part of the wave theory. Along with traditional information, the monograph contains a number of new original results that we have obtained in studying nonequilibrium and resonant phenomena in plasmalike media and that have found practical applications in electronics and radiophysics. In studying linear waves in dispersive media and systems, we proceed from the general to the special, and we hope that our theoretical study will be of interest to both beginners (students and postgraduates) and experts in the physics of wave processes. Note finally that the monograph is based

on the course of lectures given by the authors to senior students at Moscow State University (Faculty of Physics, Division of Physical Electronics).

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