NEURAL LOGIC NETWORKS

A New Class of Neural Networks

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A New Class of Neural Networks

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Contents

Preface	xi
Chapter 1	
What Are Neural Logic Networks?	1
— <i>A Gentle Introduction To The Study Of A New Class Of Neural Networks Called Neural Logic Networks</i>	
1.1 Why Do We Need A New Neural Network Model?	3
1.2 The Mathematical Definition Of Neural Logic Networks	4
1.3 A Closer Look At The Definition Of Neural Logic Networks	14
1.4 Potential Applications Of Neural Logic Networks	23
1.4.1 Potential Application #1	23
Neural Logic Networks Are Powerful Tools For The Study Of Human Logic	
1.4.2 Potential Application #2	24
Neural Logic Networks Are Powerful Tools For The Study Of 3-Valued Set Theory	
1.4.3 Potential Application #3	29
Neural Logic Networks Are Powerful Tools For The Study Of Incomplete Pattern Processing	
1.4.4 Potential Application #4	30
Neural Logic Networks Are Powerful Tools For The Study Of Knowledge Systems	
1.4.5 Potential Application #5	33
Neural Logic Networks Are Powerful Tools For The Study Of Financial Forecasts	
1.4.6 Potential Application #6	34
Neural Logic Networks Will Be Used To Integrate Symbolic Computing And Neural Computing	

1.5	The Neural Logic Family Of Networks	36
1.5.1	Abstract Neural Logic Networks	37
1.5.2	The Complex Neural Logic Networks	40
1.5.3	The 3-Valued Complex Neural Logic Networks	41
1.5.4	The 3-Valued Neural Logic Networks	41
1.5.5	The Real-Valued Neural Logic Networks	42
1.5.6	The Boolean Neural Neural Logic Networks	42
1.5.7	The Hopfield Neural Logic Networks	43
1.5.8	The Complex Hopfield Neural Logic Networks	43
1.5.9	The Probabilities And Fuzzy Logic Neural Logic Networks	44
1.5.10	The Chaotic Neural Logic Networks	47
1.5.11	Conclusion	49
1.6	The Design And Construction Of Neural Computers	49

Chapter 2

Neural Logic Networks And Human Logical Reasoning		51
2.1	What Is Human Logic?	53
2.2	Neural Logic Networks: Basic Concepts	59
2.3	Neural Logic Networks: Basic Operations	77
2.4	Neural Logic Networks: Basic Theorems	96
2.5	Propositional Neural Logic : Propositional Language	124
2.5.1	The Set Of Propositional Symbols	124
2.5.2	The Set Of Logical Operations	124
2.5.3	The Propositional Language $\mathcal{L}(\mathcal{P}, \mathcal{C})$ Generated By $\mathcal{P}$ And $\mathcal{C}$	129
2.5.4	The Propositional Algebra $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	130
2.5.5	The Propositional Logic System $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	130

2.5.6	Multi-Models Of Neural Propositional Logic Systems	131
2.5.7	Neural Logic View Of The Propositional Logic System $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	132
2.5.8	Some Basic Theorems Of $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	135
2.5.9	Counting Equivalence Classes Of $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	139
2.5.10	The Lattice-Ordered Structure Of $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	146
2.6	Propositional Neural Logic: Logical Deductions	149
2.6.1	Definitions Of Logical Deductions In $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	149
2.6.2	Formal Logical Deductions	155
2.6.3	Completeness Aspect Of The Propositional Neural Logic Systems $\langle \mathcal{L}(\mathcal{P}, \mathcal{C}), \mathcal{C} \rangle$	169
2.7	First Order Neural Logic	179
2.7.1	A Perspective Of The Proposed Theory	180
2.7.2	Step 1: The Definition Of A Universe	181
2.7.3	Step 2: Neural Logic System	189
2.7.4	Step 3: Interaction Between Logical System And Non-Logical System	194
2.7.5	Step 4: Building The First Order For The First Order Neural Logic	211
2.7.6	Step 5: The Theoretical Aspects Of The First Order Neural Logic	233
2.7.7	Step 6: The Second Order Neural Logic	243
2.8	Neural Logic: Future Research Directions	246
2.8.1	Probabilistic Neural Logic	246
2.8.2	Fuzzy Neural Logic	256
2.8.3	Inductive Neural Logic	269

2.8.4	Recurrent Neural Logic	281
2.9	Neural Logic And Human Logic	298

Chapter 3

Neural Logic And Human Intuition		301
3.1	What Is Human Intuition?	303
3.2	Neural Logic Brains	308
3.3	Neural Set Theory	329
3.4	Real World Concepts Processing	354
3.4.1	Concept Formation Game	355
3.4.2	Proposed Strategy For Concept Formation Game	364
3.4.3	Proposed Strategy For Multiple Concept Formation Game	369
3.4.4	Candidate-Elimination Strategy For Concept Formation Game	375
3.5	A Study Of Classification And Recognition Of Object Based On Attribute Patterns Processing	380
3.5.1	The Learning Algorithm Of Frank Rosenblatt	381
3.5.2	Object Classification Theorem	385
3.5.3	Pattern Matching Theorem	387
3.5.4	Higher Level Concepts: Concept of Concepts	390
3.6	Classification And Recognition Of Objects Based On Similarity Relations	403
3.6.1	Similarity Relations	403
3.6.2	Partition Trees And Partition Sequences	406
3.6.3	Equivalence Theorems Of Similarity Relations, Partition Trees And Partition Sequences	411

3.6.4	Applications	414
3.7	Classification And Recognition Of Object Based On Incomplete Attribute Patterns	416
3.7.1	Neural Logic Networks As Logic Systems	419
3.7.2	Constructions Of Effective Subsets Of Attributes	425
3.7.3	The Art Of Guessing And Testing	430
3.8	Neural Inferences	441
3.8.1	Law Of Species	441
3.8.2	Dual Species	454
3.8.3	Neural Inference Rules	462
3.8.4	Object Inference Rules And Attribute Inference Rules	467
3.8.5	Automatic Knowledge Extraction Based On Inference Rules	475
3.8.6	Multi-Dimensional Neural Inference Rules	483
3.8.7	Probabilistic Neural Inference Rules	486
3.9	Neural Logic Networks And Human Intuition -- Unfinished Tasks	497
	References	499
	Index	501

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PREFACE

This technical report is the first of a series of five reports on the following research project:

A New Model Of Neural Networks Called Neural Logic Networks

The above research project is being conducted at the Institute of Systems Science (ISS), National University of Singapore by a team of seven researchers. It is one of the key projects in the Real-World Computing Program supported by MITI of Japan.

The architecture of the project is best presented by the following diagram :

The main goal of the project is identical to the classical goal of Artificial Intelligence, namely: to model human intelligence by computing systems. What makes us believe that we may have a better chance of achieving this goal compared to other past attempts is that we are bringing in a set of new tools called neural logic networks. From a theoretical point of view, the concept of a neural logic network is a combination of three well-known concepts: a perceptron, a cellular automaton and a ring (an algebraic computational system). The fact that a neural logic network is a generalized perceptron qualifies the class of neural logic networks to be called a new class of neural networks. The fact that a neural logic network is a generalized cellular automaton implies that the class of neural logic networks is basically a new class of massively parallel machines. The reason we use an abstract algebraic structure called a ring instead of using the standard real number system (which is a special kind of ring) as the computational system for the massively parallel machines / neural networks is to increase the flexibility and versatility of our new class of tools.

With the above mentioned enhancements, we are able to develop two separate, yet interrelated, theories for the new class of neural networks. One theory shows that neural logic networks are powerful tools for emulating the human logical thinking process while the other theory shows that neural logic networks are powerful tools for emulating the basic capabilities of human intuition in the area of concept formation, concept processing, pattern recognition and classification, inductive inferencing . . . , etc.

Chapter 2 of this technical report gives a very solid development of a new type of logic called Neural Logic which is a natural extension of classical Boolean Logic. The focus point of Neural Logic is Human Logic.

In contrast to Chapter 2, Chapter 3 is an attempt to develop a theory for human intuition. However, since human intuition is harder to explicitly define or postulate than human logic, the "theory" we have is still in the stage of evolution. Future efforts are needed to bring our first-attempt theory to maturity.

When comparing the nature of human logic and human intuition we are aware of the sharp differences between their characteristics; human logic demands rigidity while human intuition demands flexibility. Human logic follows a set of well defined laws which are universal i.e. independent of individuals, locations and time. On the other hand, human intuition is very individualistic, very temporal and very subjective. Hence it is very hard to express its behaviour by universal laws. In order to capture the flexibility and dynamic nature of human intuition we introduce a new concept called the neural logic "brain". A neural logic brain is slightly different from a neural logic network in that a neural logic brain consists of a

large collection of tiny neural logic networks called neural gates. When no problem is required to be solved, these tiny neural logic networks behave like the "records" in a data base. However, when a given problem is required to be solved, many of these tiny neural gates are used to construct large neural logic networks to tackle the problem. Once the problem is solved, the large neural logic networks decompose themselves back into tiny neural gates. We believe such an approach gives a more accurate emulation of the human brain.

Neural logic networks as well as neural gates are defined as geometrical objects, because they are all defined on directed graphs. In order to be able to implement these systems on computers easily, we introduce "neural equations" which are symbolic/algebraic representations of neural gates. Hence neural gates and neural equations are the most fundamental building blocks of both neural logic networks and neural logic brains.

During the later stage of our project, we will combine the logical reasoning and pattern processing capabilities of neural logic networks to create a new methodology for the design and construction of knowledge systems. We shall call this type of knowledge systems Connectionist Knowledge Systems.

On the other hand we will develop another methodology consisting of a series of rules and techniques on how to build useful neural logic networks from tiny neural gates for real-world problem solving. Such methodology will become a kind of programming environment or a programming language. We will call this programming language Neural Prolog.

The following diagram gives a more complete perspective of our whole project :

*The architecture diagram of RWC Project :
A New Class of Neural Networks called
Neural Logic Networks*

Conducted at The Institute of Systems Science, Singapore
1993 - 1997

Members of ISS Research Team

• Professor Teh, Hoon-Heng	>> Principal Researcher
• Professor Wang, Pei -Zhuang	>> Research Advisor
• Dr Lui, Ho-Chung	>> Project Manager
• Dr. Shen, Zuliang	>> Project Coordinator - Key Researcher
• Dr. Ding, Liya	>> Key Researcher
• Mr. Lim, Joo-Hwee	>> Research Assistant
• Mr. Teow, Loo-Nin	>> Research Assistant

The following staff members of the Department of Information Systems and Computer Science (NUS) have also contributed directly or indirectly to this project :

- Associate Professor Hsu, Loke-Soo
- Associate Professor Chan, Sing-Chai
- Dr. Loe, Kia-Fock
- Dr. Tan, Chew-Lim

We would like to dedicate this technical report to Dr. Juzar Motiwalla, Director of ISS as well as all members of the ISS International Review Board for their guidance, encouragement and invaluable technical advice throughout the research process of this project.

We would also like to express our heart-felt appreciation and gratitude to Ms Lian Weeli, Admin. Manager of ISS, as well as Liz, Dora, Mary, Karen, Geok Tin ..., etc for their excellent administrative support.

This technical report is not intended to be a 'text book' or a software 'specifications'. Its main aim is to give a truthful record of our findings, whether great or small, during our course of studies of this research topic.

The principal researcher, (who is also the writer of this report), shall be responsible for all the errors and mistakes. All credits, if there is any, should go to the whole research team.

TEH HOON HENG

Principal Researcher

10th September 1993