Let Us Use
White Noise

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Preface

Why Should We?
Some Personal Comments from One Happy User.

When I embarked into the world of mathematical physics, learning about "axiomatic" quantum field theory from H. Lehmann and W. Zimmermann, and reading Borchers, Symanzik, Haag, Streater, and Wightman, I was impressed with the beauty and clarity of the LSZ and Wightman frameworks, - and quite depressed afterwards. In his book on the general theory of quantized fields, the great Res Jost wrote at the time:[1] "We had very compelling reasons for not mentioning any models except free fields. No interesting models are known ...", bad news for a junior researcher who wondered: "Will there ever be any?" And if so: "How to construct them?" Of course there were attempts; the best of them were visionary - and bad mathematics. Let me single out Feynman's "sum over histories" and the observation of Coester and Haag[2] that quantum field theory dynamics is in fact encoded in the vacuum. We knew even then that the Feynman integral was not an integral, and that the manipulations of Coester and Haag could not be justified mathematically, but it also became quite clear that there was by far not enough of infinite dimensional analysis in the physicists' mathematical tool kit. Things did get better with the physics breakthrough that goes under the name of "constructive quantum field theory", and when, on the mathematical side, 40 years ago the foundations of white noise analysis were laid.

Of course white noise analysis does not claim a monopoly: Mallavin calculus is a close relative, much like in finite dimensional analysis where there are many different Gelfand triples, suited to address particular needs. As Paul André Meyer once said in a heated debate - don't argue about the advantages of one approach or the other, show what you can do with the one that you prefer. My good friend J-A. Yan, together with Z-Y. Huang, presents the two approaches side by side in his beautiful book.[3]

So why should we use white noise analysis? Well one reason is of course that it fills that earlier gap in the tool kit. As Hida would put it, white noise provides us with a useful set of *independent* coordinates, parametrized by "time". And there is a feature which makes white noise analysis extremely user-friendly. Typically the physicist — and not only he — sits there with some heuristic ansatz, like e.g. the famous Feynman "integral", wondering whether and how this might make sense mathematically. In many cases the characterization theorem of white noise analysis provides the user with a sweet and easy answer. Feynman's "integral" can now be understood, the ansatz of Haag and Coester is now making sense via Dirichlet forms, and so on in many fields of application. There is mathematical finance, there have been applications in biology, and engineering,[4] many more than we could collect in the present volume, for some of them see e.g. Bernido and Bernido.[5]

Finally, there is one extra benefit: when we internalize the structures of Gaussian white noise analysis we will be ready to meet another close relative — we will enjoy the important similarities and differences which we encounter in the Poisson case, championed in particular by Y. Kondratiev and his group, let us look forward to a companion volume on the uses of Poisson white noise.

The present volume is essentially a collection of autonomous contributions. Fortunately however, the introductory chapter on white noise analysis was made available to the other authors early on for reference and to facilitate their efforts towards conceptual and notational coherence.

At the end of such a preface one has the right of a note of gratitude to friends and teachers. Some of the latter I have already mentioned. Then there is the "white noise community", too big by now to list it here. But I guess I have made the acquaintance and won the friendship of almost all of the white noise mathematicians you find quoted in the present volume. I also thank all of them for what they taught me. I thank the authors of the different chapters, I thank S. C. Lim of World Scientific for his invitation, help, and great patience, and can now finally, with the contributions in hand, enjoy the encouragement I got for this undertaking. May the readers enjoy those contributions and may they feel encouraged to use white noise.

Ludwig Streit, August 2016

References

1. Res Jost: The General Theory of Quantized Fields. AMS, 1965.
2. Fritz Coester and Rudolf Haag: Representation of States in a Field Theory with Canonical Variables. Phys. Rev. 117, 1137 (1960).
3. Zhi-Yuan Huang, Jia-An Yan; Introduction to Infinite Dimensional Stochastic Analysis. Springer, 2000.
4. I am thinking particular of the work of Roger Ghanem who struggled, unfortunately in vain, to meet the deadlines of this book.
5. Christopher and Victoria Bernido: Methods And Applications Of White Noise Analysis In Interdisciplinary Sciences. World Scientific, 2015.

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