CHAPTER 1

Comparative Advantage and the Assignment Problem*

Ronald W. Jones

Because the Ricardian trade model has such a simple production structure — each commodity produced only with labor at fixed coefficients — it is ideally suited to the analysis of production assignments in a world with many countries and commodities. The concept of the world transformation schedule provides the focus for our discussion here of comparative advantage in the more general case.

It is useful to consider a concrete example. Suppose the world consists of three countries: *A* (America), *B* (Britain), and *C* (Continental Europe). Furthermore suppose only three commodities can be produced and consumed: corn (*Co*), linen (*Lin*), and cloth (*Cl*). Let the numbers in the following table show the invariant labor input–output coefficients in each country to produce a unit of each commodity¹:

	<i>A</i>	<i>B</i>	<i>C</i>
Corn	10	10	10
Linen	5	7	3
Cloth	4	3	2

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¹This example, and much of the subsequent analysis, is found in R. W. Jones (1961). “Comparative Advantage and the Theory of Tariffs: A Multi-Commodity, Multi-Country Model,” *Review of Economic Studies*, June 1961, pp. 161–75, and reprinted as Chapter 3 in *International Trade: Essays in Theory*. Amsterdam: North-Holland, 1979. See also *ibid.*, Chapter 18.

Figure 1.1. The World Transformation Schedule: Three Countries

The world transformation schedule in the three-country, two-commodity version is a three-segment linear schedule bowed out from the origin. Information as to the pattern of specialization for countries A, B, C in linen and corn can be summarized in the one-dimensional Line in panel (b).

If only corn and linen are produced, Fig. 1.1 would show that the relatively most efficient producer of corn is Britain, followed by America and, last, Continental Europe, despite the fact that all three countries have the same absolute costs. In drawing Fig. 1.1, I have arbitrarily assumed all countries have the same size labor force, an assumption that does not interfere with our analysis of *patterns* of specialization in production.

Panel (b) in Fig. 1.1 summarizes much of what the two-dimensional transformation schedule in panel (a) shows. All countries produce linen at the left-hand linen origin. The lettering reveals that country B is the first country to release labor to produce corn, since it has the greatest comparative advantage in producing corn *relative to linen*. Country A is next in the

Figure 1.2. Bilateral Cost Rankings

Patterns of comparative advantage taking two commodities at a time are summarized below as in Figure 1.1(b).

line of comparative advantage so that two-thirds across to the corn origin countries *A* and *B* are completely specialized in corn and *C* in linen.

It is clear that adding one more country to a two-commodity analysis introduces no fresh difficulties. And a similar kind of ranking could display separately positions of comparative advantage in linen and cloth on the one hand and corn and cloth on the other. The information on bilateral cost ratios is computed from the table and summarized graphically in Fig. 1.2. For example, if only cloth and linen are to be produced, and if two countries are to be assigned to produce linen and only one country to produce cloth, country *B* is selected as the cloth producer because its relative cost, $\frac{3}{7}$, is lower than either $\frac{2}{3}$ or $\frac{4}{5}$. Note that there are potentially three possible assignments that put one country in cloth and two in linen. Think of a *class* of assignments as a specification of the number of countries assigned to each commodity. In this example it was zero countries in corn, one in cloth, and two in linen. (There are five other classes of this type, e.g., zero countries in cloth, one in corn, and two in linen.) The doctrine of comparative advantage singles out the *efficient* assignment in each class.

But suppose all three commodities are to be produced. If each country were to be assigned a different commodity, which assignment pattern reflects comparative advantage? Turn back to the table of cost figures. If only corn and linen were to be produced and only countries *A* and *B* involved, clearly *B* has a comparative advantage in corn and *A* in linen. Now add country *C* and the new commodity, cloth. Country *C* is in absolute terms the lowest-cost cloth producer. Therefore, suppose we consider the assignment: *A* in linen, *B* in corn, and *C* in cloth. As just noted, *A* has a

comparative advantage in linen relative to corn compared with country *B*. Likewise *C* has a comparative advantage in cloth relative to corn compared with *B* ($\frac{2}{10}$ is less than $\frac{3}{10}$). Finally, a bilateral comparison between countries *A* and *C* in linen and cloth shows that linen is produced relatively cheaply in *A* ($\frac{5}{4}$ is smaller than $\frac{3}{2}$). And yet this pattern of specialization is inefficient! There are six possible assignments in which all three commodities are produced and the efficient one has corn produced by country *A*, cloth by country *B*, and linen by country *C*.

To check out that this alternative assignment is indeed efficient, recall that the force of competition in world markets rules out inefficient production patterns. In a competitive equilibrium, any commodity that is actively produced in a country must have unit labor costs equal to price so that profits are bid away. Furthermore, if a country does not produce a commodity, it must not be the case that unit labor costs are actually less than the world price of that commodity. As we now show, there is a range of world prices at which, simultaneously, country *A* can produce corn, country *B* cloth, and country *C* linen.

Since only relative prices matter, let us arbitrarily set the price of corn at unity. If country *A* is to produce corn, the wage rate in country *A* must equal $\frac{1}{10}$. This, in turn, reveals that if country *A* were to attempt to produce linen, unit costs would equal $\frac{1}{2}$. Therefore, a competitive equilibrium in which country *A* does not produce linen must be one in which the world price of linen, p_{Lin} , is less than or equal to $\frac{1}{2}$. Figure 1.3 shows possible ranges for the world prices of linen and cloth (given our assumption that the price of corn is unity). Any value of p_{Lin} greater than $\frac{1}{2}$ is thus ruled out. Also, with a wage rate of $\frac{1}{10}$ the unit cost of producing cloth in *A* would be $\frac{2}{5}$, and this cannot fall short of the world price of cloth.

Now consider country *B*, which has been assigned to cloth production. The wage rate in country *B* depends on the price of cloth, and will equal one-third times p_{Cl} . Therefore, if *B* were to attempt to produce corn, the cost per unit would be 10 times the wage rate, which must not fall short of the price of corn (which equals unity by assumption). That is, $\frac{10}{3}p_{Cl}$ must be ≥ 1 , which puts a lower bound on the price of cloth. Similarly, unit costs in *B* for linen production would equal $(\frac{7}{3})p_{Cl}$, which must be greater than or equal to the world price of linen. That is, any acceptable p_{Lin} must lie below the ray from the origin whose equation is $p_{Lin} = \frac{7}{3}p_{Cl}$. Finally, consider the required comparison in country *C* between the unit costs of corn and cloth

Figure 1.3. Price Possibilities

If the price of corn is unity, any price combination for linen and cloth in the shaded area satisfies the competitive profit conditions when *A* produces corn, *B* cloth, and *C* linen.

and world prices. *C* is assigned to linen, so that the wage rate in *C* equals one-third the world price of linen. Its unit costs in corn would then be $\frac{10}{3}$ times the price of linen so that $\frac{10}{3} P_{Lin}$ must exceed unity to avoid *C* being a potentially profitable location for corn production. Finally, unit costs for cloth in *C* would be two times the wage rate in *C* or $\frac{2}{3}$ times the price of linen. This cost figure must not fall short of the price of cloth, which means P_{Lin} must exceed $\frac{3}{2}$ times the price of cloth.

The shaded area in Fig. 1.3 shows combinations of prices of linen and cloth, relative to a unit price for corn, at which competition allows America to produce corn, Britain to produce cloth, and, simultaneously, Continental Europe to produce linen. Of the six assignments in the *class* for which each country produces a commodity distinct from the other two, this one is the only efficient one. There is *no* set of free-trade prices that would tolerate the others (e.g., *A* in linen, *B* in corn, and *C* in cloth as we originally suggested). This information is necessary in drawing the full world transformation surface. Such a surface would be three-dimensional, but a two-dimensional representation of the assignments can be made, just as panel (b) of Fig. 1.1 contained in one dimension the information about assignments captured in panel (a)'s transformation locus.

Figure 1.4. Assignments Along the World Transformation Surface

Along the ridges one country's labor force is being reallocated between two of the three commodities.

Figure 1.4 shows the efficient assignment pattern along the world transformation surface.² The bottom edge of the triangle corresponds to panel (b) of Fig. 1.1, while the side edges replicate the assignment patterns in Fig. 1.2. In constructing the shape of the surface it was necessary to go through our previous analysis in search of the efficient assignment for the interior point where all three commodities are produced and each country is specialized. Figure 1.4 suggests that the transformation surface is made up of planar elements, ridges, and points. Points correspond to positions in which all countries are completely specialized, and once the efficient assignment in each class of complete specialization is determined, the entire surface can be mapped out. Suppose we start at the linen corner, where all countries are producing linen. The cost figures suggest that country *B* has a strong comparative disadvantage in linen production, so that should the world wish to consume small amounts of either cloth or corn (or both), it is country *B*'s labor that is first freed up from its assignment to linen. Indeed, relative commodity prices in the lower left-hand triangle in Fig. 1.4

²The technique of reducing a three-dimensional surface to a two-dimensional pattern of assignments was pioneered by Lionel McKenzie in "Specialization and Efficiency in World Production," *Review of Economic Studies*, June 1954.

are given solely by comparative labor costs in country B . Thus if the price of corn is unity, the price of linen would be $\frac{7}{10}$ and of cloth $\frac{3}{10}$. The lower left-hand triangle exactly replicates country B 's planar transformation surface. At a point such as x , country B is completely specialized in cloth, while country A produces both linen and corn and country C produces linen and cloth. Thus country A 's technology determines the linen/corn relative price ($\frac{1}{2}$) and country C 's labor costs determine that the relative price of linen to cloth is $\frac{3}{2}$. Along a ridge just one country is incompletely specialized to two commodities, so that one relative price is fixed and the other free to vary (within bounds). At the interior point showing complete specializations, both relative prices are free to vary within the bounds set by the three adjacent planes; this corresponds to relative prices in the shaded area of Fig. 1.3.

There is a simple general rule for locating the assignment that is efficient in its class. Reconsider the interior class in which each country is specialized completely to a different commodity. In the efficient specialization A was assigned corn, B cloth, and C linen so that in symbols,

$$\begin{aligned}w^A a_{Co}^A &= p_{Co} \\w^B a_{Cl}^B &= p_{Cl} \\w^C a_{Lin}^C &= p_{Lin},\end{aligned}$$

where w^i is country i 's wage rate and a_j^i is country i 's labor cost of producing a unit of commodity j . It is also the case that

$$\begin{aligned}w^A a_{Lin}^A &\geq p_{Lin} \\w^B a_{Co}^B &\geq p_{Co} \\w^C a_{Cl}^C &\geq p_{Cl}\end{aligned}$$

since otherwise some country would have an unexploited profitable production possibility. Multiply the three equalities to get:

$$w^A w^B w^C a_{Co}^A a_{Cl}^B a_{Lin}^C = p_{Co} p_{Cl} p_{Lin}.$$

Multiply the three inequalities, with at least one strict inequality, to obtain:

$$w^A w^B w^C a_{Lin}^A a_{Co}^B a_{Cl}^C > p_{Co} p_{Cl} p_{Lin}.$$

A comparison now reveals that if the assignment $A-Co$, $B-Cl$, $C-Lin$ was the efficient member of its class, the product $a_{Co}^A a_{Cl}^B a_{Lin}^C$ must not exceed

$a_{Lin}^A a_{Co}^B a_{Cl}^C$. According to the table, the first product is 90 while the second is 100. The general rule is that the assignment pattern that *minimizes the product of labor coefficients* among all assignments in its class is the efficient optimal assignment. No other assignment patterns in that class can ever be supported in a free-trade equilibrium. This is the generalization of the Ricardian law of comparative advantage that in the two-commodity, two-country case was cast in terms of cost ratios.

As a final observation note that the doctrine of comparative advantage does not state what a country *will* produce. The actual production pattern depends largely upon demand. Instead, it points out production patterns that are *ruled out* by free trade. In our 3×3 numerical example there are 3^3 or 27 possible complete specialization assignments of countries to commodities. Of these 17 are inefficient and would never be observed in a free-trade world. The remaining 10 are the ones shown in Fig. 1.4. The fraction of inefficient production patterns ruled out by the austere pressures of competition and free trade rises drastically as the number of countries and commodities expands.³

³For further discussion see R. W. Jones, *International Trade: Essays in Theory* (Amsterdam: North-Holland, 1979), Chapter 18.