Combinatorial Problems in Mathematical Competitions

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Introduction

This book consists of three parts: fundamental knowledge, basic methods and typical problems. These three parts introduce the fundamental knowledge of solving combinatorial problems, the important solutions to combinatorial problems and some typical problems with often-used solutions in the high school mathematical competition respectively.

In each chapter there are necessary examples and exercises with solutions. These examples and exercises are of the same level of difficulty as the China Mathematical League Competitions which are selected from mathematical competitions at home and abroad in recent years. Some test questions are created by the author himself and a few easy questions in China Mathematical Olympiad (CMO) and IMO are also included. In this book, the author pay attention to leading readers to explore, analyze and summarize the ideas and methods of solving combinatorial problems. The readers' mathematical concepts and abilities will be improved remarkably after acquiring knowledge from this book.

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Preface

Combinatorial Mathematics has a long history. Thousands of years ago, some simple and interesting combinatorial problems started to be involved in the ancient Chinese masterpieces such as *He Tu* and *Luo Shu*. In the recent 20 years, owing to the rapid development of knowledge such as Computer Science, Coding Theory, Programming Theory, Digital Communication and Experimental Designing, a series of theoretical and practical problems need to be solved by discrete mathematics. Moreover, the questions which were raised by the logic of combinatorial mathematics itself and other branches of mathematics, has made the research about combinatorial mathematics to flourish and very rich. The problem-solving techniques and methods are daedal, transforming this ancient mathematical idea into a rigorous mathematical subject.

The combinatorial questions in the mathematical competitions are always straightforward, but the people who solve these problems should be endowed with the power of acute observation, rich imagination and necessarily skills. There are no fixed methods that can be followed, and various questions of different level of difficulties are very rich. Hence, combinatorial questions have been included in different levels of intelligence test and mathematical competition.

This book focus on the combinatorial problems in the mathematics competition, and consists of three parts: fundamental knowledge, basic methods and typical problems. This book is a translation from the author's book of the same title (in Chinese), with a few amendments

and supplements. It not only emphasizes the fundamental knowledge of solving combinatorial problems in mathematical competitions, but also introduces to the readers the important solutions to combinatorial problems and some typical problems with solutions that are often used.

In the choice of examples and exercises, except the questions which strikingly newer have been selected as far as possible, but the novel test questions which are the same difficult level as the China Mathematical League Competitions in the mathematical competition at home and abroad in recent years have been chosen deliberately. Some test questions created by the author himself and a few easy questions in China Mathematical Olympiad (CMO) and IMO are also included. In this book, the author pays attention to leading the readers to explore, analyze and summarize the ideas and methods of solving combinatorial problems. The readers' mathematical concepts and abilities will be improved remarkably after acquiring the knowledge from this book.

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