Oscillation Theory of Partial Differential Equations

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Norio Yoshida

University of Toyama, Japan

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World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN-13 978-981-283-543-7

ISBN-10 981-283-543-1

Printed in Singapore.

To my family
Chikiko,
Aki, Hikari, Sachi, and Hidemasa

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Preface

Oscillation theory of differential equations, originated from the monumental paper of C. Sturm published in 1836, has now been recognized as an important branch of mathematical analysis from both theoretical and practical viewpoints.

Roughly speaking, the objective of oscillation theory is to acquire as much information as possible about the qualitative properties of solutions of differential equations through the analysis of laws governing the distribution of zeros of solutions as well as the asymptotic behavior of solutions of differential equations under consideration.

Oscillation theory has been attracting wide attention of researchers for the past five decades, and as a consequence of their great efforts it has grown up to be a fertile field covering ordinary differential equations, functional differential equations with deviating arguments, partial differential equations with or without functional arguments and difference equations.

It goes without saying that a vast literature, research papers and books, dealing with oscillation theory has been published so far. As regards books and monographs on the subject, most of the existing ones are exclusively concerned with ordinary differential equations and/or difference equations, and there seems to be none which is devoted to the study of oscillation of partial differential equations.

The present book was motivated by this observation and is designed to provide the reader with an exposition of interesting aspects, from rudimentary to advanced, of oscillation theory of partial differential equations which dates back to the publication in 1955 of a paper by Ph. Hartman and A. Wintner.

The main body of this book is divided into eight chapters. In the first three chapters basic oscillation theory is developed for the standard three

types of partial differential equations without functional arguments, namely, elliptic equations (Chapter 1), parabolic equations (Chapter 2) and hyperbolic equations (Chapter 3). After discussing oscillation of a class of fourth order Beam equations in the short Chapter 4, we will be concerned with partial differential equations with functional arguments in the following three chapters; more specifically, we present modern oscillation results for functional elliptic equations, functional parabolic equations and functional hyperbolic equations in Chapters 5, 6 and 7, respectively. In the final Chapter 8 we establish Picone type identities which generalize the celebrated identity of M. Picone discovered in 1909, and show that they can be effectively applied to the study of oscillation of a variety of partial differential equations involving half-linear elliptic equations and quasi-linear parabolic equations and systems. The multi-dimensional Riccati method is also investigated in connection with Picone type identities. It should be emphasized that in order to circumvent the difficulties of multi-dimensional analysis the oscillation problem for partial differential equations is often reduced to that of one-dimensional differential equations or inequalities, which is easier to handle, so that knowledge of oscillation theory of ordinary differential equations is indispensable to the investigation of oscillation properties of partial differential equations.

It is hoped that this book will be a standard textbook on oscillation theory of partial differential equations which is useful for both specialists and graduate students working in the field of differential equations, and also that the book will help stimulate further progress in the study of oscillation theory and related subjects.

I am greatly indebted to Professor T. Kusano for drawing my attention to the attractive subject of oscillation theory of partial differential equations and for his continual guidance and encouragement. I wish to express my sincere gratitude to Professor H. Onose for recommending me to write a book on multi-dimensional oscillation theory. Thanks are also due to my friend Professor M. Naito for many stimulating and beneficial discussions on oscillation problems as well as to my collaborators including Professors J. Jaroš, T. Kiguradze, K. Kreith, J. Manojlović, E. S. Noussair, Y. Shoukaku, S. Tanaka, T. Tanigawa, Y.-S. Tao and L.-T. Yan. Finally, I would like to express my heartfelt thanks to Professor K K Phua, Chairman of World Scientific Publishing Company, for his great consideration, and to Editor Ms. Tan Rok Ting for her expert editing of the manuscript.

N. Yoshida

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