NUMERICAL SIMULATION OF WAVES AND FRONTS IN INHOMOGENEOUS SOLIDS

WORLD SCIENTIFIC SERIES ON NONLINEAR SCIENCE

Editor: Leon O. Chua
University of California, Berkeley

Series A. MONOGRAPHS AND TREATISES*

- Volume 45: Bifurcation and Chaos in Nonsmooth Mechanical Systems
J. Awrejcewicz & C.-H. Lamarque
- Volume 46: Synchronization of Mechanical Systems
H. Nijmeijer & A. Rodriguez-Angeles
- Volume 47: Chaos, Bifurcations and Fractals Around Us
W. Szemplińska-Stupnicka
- Volume 48: Bio-Inspired Emergent Control of Locomotion Systems
M. Frasca, P. Arena & L. Fortuna
- Volume 49: Nonlinear and Parametric Phenomena
V. Damgov
- Volume 50: Cellular Neural Networks, Multi-Scroll Chaos and Synchronization
M. E. Yalcin, J. A. K. Suykens & J. P. L. Vandewalle
- Volume 51: Symmetry and Complexity
K. Mainzer
- Volume 52: Applied Nonlinear Time Series Analysis
M. Small
- Volume 53: Bifurcation Theory and Applications
T. Ma & S. Wang
- Volume 54: Dynamics of Crowd-Minds
A. Adamatzky
- Volume 55: Control of Homoclinic Chaos by Weak Periodic Perturbations
R. Chacón
- Volume 56: Strange Nonchaotic Attractors
U. Feudel, S. Kuznetsov & A. Pikovsky
- Volume 57: A Nonlinear Dynamics Perspective of Wolfram's New Kind of Science
L. O. Chua
- Volume 58: New Methods for Chaotic Dynamics
N. A. Magnitskii & S. V. Sidorov
- Volume 59: Equations of Phase-Locked Loops
J. Kudrewicz & S. Wasowicz
- Volume 60: Smooth and Nonsmooth High Dimensional Chaos and the Melnikov-Type Methods
J. Awrejcewicz & M. M. Holicke
- Volume 61: A Gallery of Chua Attractors (with CD-ROM)
E. Bilotta & P. Pantano
- Volume 62: Numerical Simulation of Waves and Fronts in Inhomogeneous Solids
A. Berezovski, J. Engelbrecht & G. A. Maugin

*To view the complete list of the published volumes in the series, please visit:
http://www.worldscibooks.com/series/wssnsa_series.shtml

Series Editor: Leon O. Chua

NUMERICAL SIMULATION OF WAVES AND FRONTS IN INHOMOGENEOUS SOLIDS

Arkadi Berezovski
Jüri Engelbrecht

Tallinn University of Technology, Estonia

Gérard A Maugin

Université Pierre et Marie Curie, France

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Berezovski, Arkadi.

Numerical simulation of waves and fronts in inhomogeneous solids / by Arkadi Berezovski, Jüri Engelbrecht, Gérard A Maugin.

p. cm. -- (World Scientific series on nonlinear science. Series A ; v. 62)

Includes bibliographical references and index.

ISBN-13: 978-981-283-267-2 (hardcover : alk. paper)

ISBN-10: 981-283-267-X (hardcover : alk. paper)

1. Elastic solids. 2. Inhomogeneous materials. 3. Wave motion, Theory of.

I. Engelbrecht, Jüri. II. Maugin, G. A. (Gérard A.), 1944- III. Title.

QA935.B375 2008

530.4'12--dc22

2008015670

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 2008 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

Printed in Singapore.

Preface

Numerical simulations of dynamical problems in inhomogeneous solids are quite difficult for the case of moving discontinuities such as phase transition fronts or cracks. The origin of these difficulties is a constitutive deficiency in the thermomechanical description of the corresponding irreversible processes, which leads to an uncertainty in jump relations at moving discontinuities. Consequently, the construction of an appropriate numerical algorithm should be complemented by the development of the thermomechanical theory.

The aim of this book is to provide a framework for the description of moving discontinuities in solids and its implementation in a finite-volume numerical algorithm.

The macroscopic description of moving discontinuities in solids (such as phase-transition fronts or crack fronts) needs to go beyond the conventional local equilibrium approximation, since both phase-transition fronts and cracks propagate irreversibly accompanied by entropy production at their moving front.

The numerical simulation of thermomechanical processes supposes the application of a certain discretization method. Therefore, instead of infinitesimally small material “points”, which are in local equilibrium, in numerical simulations we deal with finite-size computational cells, states of which are non-equilibrium ones in general. To keep the meaning of thermodynamic quantities well-defined, a projection of the non-equilibrium states onto an equilibrium subspace is used conventionally. This means that the non-equilibrium state of each computational cell is associated with an equilibrium state of the accompanying reversible process. The simplest projection is the averaging of corresponding fields over the computational cell. An averaging procedure over the control volume applied to appropriate conser-

vation laws leads to a system of equations in terms of averaged quantities and corresponding fluxes at boundaries of cells. Such a procedure is a basis for Godunov-type finite-volume numerical schemes.

The main problem in the construction of a particular algorithm is the proper determination of numerical fluxes. However, even if we are able to determine the numerical fluxes in the bulk, their determination at moving fronts is highly questionable.

Throughout the book, all needed fluxes are determined by means of the local equilibrium jump relations. These local equilibrium jump relations are formulated in terms of excess quantities, which characterize the non-equilibrium states of computational cells. The continuity of excess quantities across the moving discontinuity is used for the closure of the model. This leads to the relation between the stress jump at the discontinuity and the corresponding driving force, which allows us to determine the velocity of a moving front. As a consequence, a thermodynamically consistent finite volume numerical algorithm is obtained, which can be applied to both wave and front propagation problems.

It is remarkable that similar considerations are applicable to another problem of moving discontinuity, namely, to crack dynamics. In the case of a moving crack, the local equilibrium jump relation at the crack front is complemented by another assumption regarding the excess quantities, which reflects the distinction between the problems of moving phase boundaries and cracks.

Examples of numerical simulations of phase-transition front propagation and straight brittle crack motion show the applicability of the developed non-equilibrium description to solve the problems of moving fronts.

A. Berezovski, J. Engelbrecht, G.A. Maugin

Contents

<i>Preface</i>	v
1. Introduction	1
1.1 Waves and fronts	1
1.2 True and quasi-inhomogeneities	2
1.3 Driving force and the corresponding dissipation	2
1.4 Example of a straight brittle crack	3
1.5 Example of a phase-transition front	4
1.6 Numerical simulations of moving discontinuities	5
1.7 Outline of the book	7
2. Material Inhomogeneities in Thermomechanics	9
2.1 Kinematics	9
2.2 Integral balance laws	11
2.3 Localization and jump relations	12
2.3.1 Local balance laws	12
2.3.2 Jump relations	13
2.3.3 Constitutive relations	14
2.4 True and quasi-material inhomogeneities	14
2.4.1 Balance of pseudomomentum	15
2.5 Brittle fracture	18
2.5.1 Straight brittle crack	20
2.6 Phase-transition fronts	23
2.6.1 Jump relations	23
2.6.2 Driving force	24

2.7	On the exploitation of Eshelby's stress in isothermal and adiabatic conditions	26
2.7.1	Driving force at singular surface in adiabatic conditions	28
2.7.2	Another approach to the driving force	29
2.8	Concluding remarks	30
3.	Local Phase Equilibrium and Jump Relations at Moving Discontinuities	31
3.1	Intrinsic stability of simple systems	32
3.2	Local phase equilibrium	34
3.2.1	Classical equilibrium conditions	34
3.2.2	Local equilibrium jump relations	36
3.3	Non-equilibrium states	38
3.4	Local equilibrium jump relations at discontinuity	39
3.5	Excess quantities at a moving discontinuity	41
3.6	Velocity of moving discontinuity	43
3.7	Concluding remarks	44
4.	Linear Thermoelasticity	45
4.1	Local balance laws	45
4.2	Balance of pseudomomentum	47
4.3	Jump relations	47
4.4	Wave-propagation algorithm: an example of finite volume methods	48
4.4.1	One-dimensional elasticity	48
4.4.2	Averaged quantities	50
4.4.3	Numerical fluxes	51
4.4.4	Second order corrections	52
4.4.5	Conservative wave propagation algorithm	52
4.5	Local equilibrium approximation	53
4.5.1	Excess quantities and numerical fluxes	53
4.5.2	Riemann problem	55
4.5.3	Excess quantities at the boundaries between cells	56
4.6	Concluding remarks	57
5.	Wave Propagation in Inhomogeneous Solids	59
5.1	Governing equations	60

5.2	One-dimensional waves in periodic media	60
5.3	One-dimensional weakly nonlinear waves in periodic media	62
5.4	One-dimensional linear waves in laminates	65
5.5	Nonlinear elastic wave in laminates under impact loading	68
5.5.1	Problem formulation	70
5.5.2	Comparison with experimental data	71
5.5.3	Discussion of results	77
5.6	Waves in functionally graded materials	80
5.7	Concluding remarks	83
6.	Macroscopic Dynamics of Phase-Transition Fronts	85
6.1	Isothermal impact-induced front propagation	87
6.1.1	Uniaxial motion of a slab	88
6.1.2	Excess quantities in the bulk	90
6.1.3	Excess quantities at the phase boundary	91
6.1.4	Initiation criterion for the stress-induced phase transformation	93
6.1.5	Velocity of the phase boundary	97
6.2	Numerical simulations	100
6.2.1	Algorithm description	100
6.2.2	Comparison with experimental data	100
6.3	Interaction of a plane wave with phase boundary	102
6.3.1	Pseudoelastic behavior	104
6.4	One-dimensional adiabatic fronts in a bar	109
6.4.1	Formulation of the problem	109
6.4.2	Adiabatic approximation	112
6.4.3	Initiation criterion for the stress-induced phase transformation in adiabatic case	112
6.4.4	Velocity of the phase boundary	115
6.4.5	Temperature field	117
6.5	Numerical simulations	118
6.5.1	Pulse loading	119
6.5.2	Temperature distribution	120
6.5.3	Kinetic behavior	122
6.6	Concluding remarks	125
7.	Two-Dimensional Elastic Waves in Inhomogeneous Media	127
7.1	Governing equations	128

7.1.1	Averaged quantities	128
7.1.2	Conservation law	129
7.2	Fluctuation splitting	130
7.3	First-order Godunov scheme	133
7.4	Transverse propagation	135
7.4.1	Vertical transverse propagation	135
7.4.2	Horizontal transverse propagation	137
7.4.3	Boundary conditions	139
7.5	Numerical tests	139
7.6	Concluding remarks	145
8.	Two-Dimensional Waves in Functionally Graded Materials	147
8.1	Impact loading of a plate	147
8.2	Material properties	149
8.3	Numerical simulations	151
8.4	Centreline stress distribution	153
8.5	Wave interaction with functionally graded inclusion . . .	155
8.6	Concluding remarks	157
9.	Phase Transitions Fronts in Two Dimensions	161
9.1	Material velocity at the phase boundary	161
9.2	Numerical procedure	163
9.3	Interaction of a non-plane wave with phase boundary . . .	164
9.4	Wave interaction with martensitic inclusion	166
9.5	Concluding remarks	168
10.	Dynamics of a Straight Brittle Crack	171
10.1	Formulation of the problem	172
10.2	Stationary crack under impact load	173
10.3	Jump relations at the crack front	175
10.4	Velocity of the crack in mode I	176
10.4.1	Zero excess stress	178
10.4.2	Non-zero excess stress	179
10.5	Concluding remarks	182
11.	Summing Up	185
Appendix A	Thermodynamic interaction between two discrete systems in non-equilibrium	189

A.1	Equilibrium/non-equilibrium contacts	190
A.1.1	Contact quantities	190
A.1.2	Partial contact quantities	193
A.2	Interacting non-equilibrium systems	195
A.2.1	Replacement quantities	195
A.2.2	Composite systems	197
A.3	Compound deficiency	200
A.3.1	The inequalities	200
A.3.2	Energy and entropy	201
A.3.3	Example: An endoreversible system	203
A.3.4	Excess quantities	205
A.4	Concluding remarks	206
	<i>Bibliography</i>	209
	<i>Index</i>	221