Eternal Inflation

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Printed in Singapore.

To my grandparents, to my true teachers, and to Mylène

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Preface

This book is an introduction to the theory of “multiverse” in inflationary cosmology and to the so-called “measure problem.”

It is likely, given the present state of cosmological knowledge, that the universe may continue expanding forever and that local conditions in the universe are at least to some extent randomly produced. As a result, different and causally independent regions of the universe — that is to say, regions inaccessibly far removed from each other in space and time — may, by chance, acquire the hot homogeneous state (the “hot Big Bang”) that is known to be the origin of our observed universe. It follows that, given unlimited time, the universe will generate an *infinite* number of regions similar to ours, where galaxies and stars may give rise to intelligent life capable of scientific observation. Each region where local conditions are approximately homogeneous is called a pocket universe, while the totality of (infinitely many) pocket universes is called a multiverse.

In almost all scenarios of multiverse cosmology (the most recent such scenario being the “landscape” of string theory vacua) it is virtually certain that the different pocket universes will have different physical properties, such as the observed values of physical constants, including the masses of elementary particles, the fine structure constant, and the effective value of the cosmological constant. Since our present location in the universe is (presumably) in no way special, it follows that the presently observed values of physical constants are randomly generated and cannot be predicted on the basis of a fundamental “theory of everything.” In other words, the local values of “constants of nature” in each pocket universe are *environmental givens* rather than manifestations of global underlying physical properties.

A natural approach in this situation is to try predicting the probability

distribution for the local values of physical constants observed in a randomly chosen pocket universe. This approach has been initiated in the late 1980s, and quite soon a difficulty (now known as the measure problem) presented itself. The root of the measure problem is the fact that it is impossible to compute statistical averages directly on the ensemble of infinitely many pocket universes. The notion of a “randomly chosen” pocket universe is mathematically ill-defined. One needs to introduce a cutoff on the infinite set of pocket universes; however, the results depend sensitively on the cutoff procedure.

The measure problem has resisted solution for more than two decades since its original formulation in the early 1990s, which was in the context of cosmological inflation driven by scalar fields. At present, the renewed interest in the measure problem is due to the discovery of inflationary models in the string-theoretic landscape, which allows a very large number of metastable vacua with different physical laws. The need to extract predictions from these models has stimulated significant research activity in this field, resulting in the appearance of several competing measure proposals and a deeper understanding of the issues involved.

This book gives an overview of the presently known approaches to the measure problem and proceeds to describe a novel measure proposal, which was developed in a series of recent publications. Currently the measure problem in multiverse cosmology is an active area of research, and more progress may be achieved in the near future. Rather than trying to anticipate the forthcoming results, I wish to concentrate on the exposition of concepts and mathematical methods that will probably remain useful for future research in this area. I give merely a brief expository presentation of inflationary cosmology, referring the reader to numerous available textbooks for the needed background material. To recover the necessary background, the reader is only obliged to follow an introductory course in inflationary cosmology and cosmological perturbations. Similarly, I include only a necessary minimum of facts regarding the landscape of string theory; no knowledge of string theory is assumed or needed in this book. On the other hand, I show some derivations and illuminate details not ordinarily given in the literature on eternal inflation. Terminology defined in this text is shown in **boldface**; I reserve *italics* for emphasis.

The first research papers on eternal inflation was published by Andrei Linde, Alexey Starobinsky, and Alexander Vilenkin between 1982 and 1986.

I will attempt to give due credit by referencing substantially all contributors to the field. A recent book [Vilenkin (2006)] tells the story of eternal inflation in a form accessible to the general public. The present book is an introduction to the technical side of that story.

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Sergei Winitzki, March 2009

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