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The Richard Rand 50th Anniversary Volume

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Professor Rand at the blackboard. (Photo by Professor C. Orloff.)

Dedication

This volume of *Stability, Vibration, and Control of Structures* consists of a collection of articles, dedicated to Richard Rand, Professor of Theoretical and Applied Mechanics at Cornell, on the occasion of his fiftieth birthday on June 8, 1993.

Professor Richard Rand was born and raised on Manhattan Island in New York City. As a boy he showed an early interest in science, especially electronics and amateur radio. He graduated from the Bronx High School of Science, from Cooper Union and finally got his doctorate at Columbia University in the Department of Civil Engineering and Engineering Mechanics in 1967. In those days Columbia had one of the strongest programs in the country in applied mechanics, and Rand took courses from such famous professors as Raymond Mindlin, Hans Bleich, Bruno Boley, Dick Skalak and Frank DiMaggio, who was his doctoral thesis adviser. Rand's thesis was on the linear vibrations of fluid-filled prolate spheroidal shells.

He joined the faculty of Cornell in the Department of Theoretical and Applied Mechanics (TAM) in September of 1967 as an Assistant Professor, only a few weeks after receiving his doctorate and at the early age of twenty-four. He has continued that position to the present day, having been duly promoted to Associate and Full Professor. His research in dynamics has involved fruitful interactions with Cornell Professors Terry Alfried, Joe Burns, Frank Moon, Phil Holmes and John Guckenheimer, to name a few. His interest in computer algebra has resulted in interactions with Professor Dieter Armbruster of the University of Arizona, and with Professor Moss Sweedler of Cornell. His work at Cornell has included numerous projects in bioengineering with Professor Bob Cooke of the Department of Agricultural Engineering, as well as with Professor H. David Block of TAM, and Dr. Avis Cohen and Professor Howard Howland of the Department of Neurobiology and Behavior.

He enjoyed a sabbatic in 1981-82 in the Department of Mechanical Engineering at the University of California at Berkeley, where he co-taught a course on advanced dynamics with Professors R. M. Rosenberg, C. S. Hsu and Leslie Anne Month, and in 1989-90 in the Department of Mechanical, Aerospace and Nuclear Engineering at the University of California at Los Angeles, where he collaborated with Professors Tino Mingori and Bill Newman.

Professor Richard Rand is an outstanding teacher. His classroom delivery is unusually clear and insightful, and he provides assignments that are designed to take the students into the issues and ideas beyond those seen in the classroom. Many students found his teaching style to be inspiring, largely due to his ability to get to the essence of a concept and place it in a tractable setting, and explain it simply and correctly. In addition he has been a national leader in bringing computer-aided symbolic processing programs into the classrooms, at both graduate and undergraduate levels. He has been using symbolic processors in his teaching and research for over fifteen years, and has set up a very effective system at Cornell for using these programs in mechanics and mathematics courses. These efforts have been in part responsible for the two awards he has won for excellence and innovation in teaching. His presentation skills are also evident when giving research seminars at professional meetings, where he invariably gives one of the best talks in a symposium.

While Professor Rand's teaching accomplishments are impressive, it would be misleading to suggest that they outweigh his contributions in research. He has made significant contributions to the literature of theoretical and applied mechanics. He is the author of three books, and has published more than one hundred journal papers. His technical work addresses significant contributions in biomechanics, nonlinear dynamics, satellite dynamics and applied mathematics. In biomechanics he contributed to the biomechanics of green plants, including gas exchange, fluid flow in stem and leaf, stomatal dynamics and circumnutation; he also contributed to neurobiology of swimming in lamprey, and modeling of the human eye, including retinal dynamics and corneal mechanics. In nonlinear dynamics he contributed to the understanding of "nonlinear normal modes" and investigated criteria for n limit cycle oscillators to move in unison. His work on nonlinear normal modes

followed the ground-breaking work of Professor R.M. Rosenberg by obtaining many important results on the stability and bifurcation of normal modes of vibration when energy levels become moderately large. These results are especially important when the system possesses internal resonances, as bifurcations occur which lead to dynamics that are significantly different from those predicted by linear theory. He applied the idea of nonlinear normal modes to the problem of fruit harvesting, specifically to develop methods of tree shaking which enhance the removal of fruit from the tree, either with, or without, the stem (depending upon which mode is excited.) In satellite dynamics, he studied nonlinear effects near the “triangular points” in the restricted 3 body problem, as well as the phenomena of “resonant capture” and “frequency demultiplication”. His results on resonance capture have important implications for the performance of any device which must, during transient periods such as start-up, pass through a vibratory resonance of the system. He successfully used these methods to describe and predict the potentially disastrous resonant capture that can occur during spin-up of axial, dual-spin spacecraft. Professor Rand also contributed to our understanding of chaotic motion of pipes conveying fluid, space structures with pinned joints, and periodic loading of elastic columns. In applied mathematics he investigated the use of elliptic functions in averaging. He also applied computer algebra to the implementation of modern perturbation schemes such as center manifolds, normal forms, and Lie transforms on digital computers. In addition he has written a variety of papers on such diverse subjects as wheel shimmy in motorcycles, thermal stresses in chicken eggs during commercial washing, Hilbert’s 16th problem, determinacy of singular vector fields, bifurcation of limit cycles from infinity in the phase plane, parametric stiffness control of flexible structures. In each line of research, he makes extensive use of computer-assisted symbolic processing. Over the past years he has developed a comprehensive set of computer algebra/perturbation method tools in the form of programs which have been summarized in three clearly-written books on the topic.

In the lives of most of us, there is often one person, frequently one of our teachers, who does more than others to guide our future careers and to inspire us to take on challenging and difficult tasks. In the career of many of us, this all important contact was Professor Rand, a contributor to many aspects of theoretical and applied mechanics and an outstanding and inspirational

teacher. He has been very helpful to many young scholars. Not only he has produced more than a dozen Ph.D. students, many of whom have obtained positions at prestigious universities, but he has also played an important role in the development of others. His close to seventy collaborators and those individuals from meetings and lecture halls who have met Richard know very well the high value he places on open non-adversarial discussions and on intellectual curiosity.

In the spirit of a birthday celebration, the participants join with Professor Rand in celebrating the enjoyment we all share in doing research. In this volume, each contributor describes some of their research, as we all look forward to many more years of problem solving and of sharing our results. We dedicate this volume to our friend and mentor Richard Rand “with the feeling of excitement that we all have at the beginning of a new problem, yet unsolved, and with it the chance to use our craft”.

I gratefully acknowledge the help of Professors Steven Shaw (Michigan State), Brian Fenny (Michigan State), Jacquelyn Bridge (Georgie Tech), Christopher Hall (Air Force Institute of Technology), Tim Healey (Cornell), Joseph Cusumano (Pen State), Edwin Kreuzer (Tech. Univ. Hamburg-Harburg), Andy Ruina (Cornell), Alexander Vakakis (Univ. Illinois Urbana-Champaign), Oliver O'Reilly (U.C. Berkeley), Larry Bergman (Univ. Illinois Urbana-Champaign), Daniel Inman (VPISU), Anil Baja (Purdue University), Raymond Plaut (VPISU), Dr. Sharon Lubkin (U. Washington), in providing materials for this homage.

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