Oscillations in Planar Dynamic Systems

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Oscillations in Planar Dynamic Systems

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This book is dedicated to the memory of
Professor Henry C. McBay (1914–1995),
a gentle man,
teacher, scholar, researcher,
and friend.

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PREFACE

This work is a complete revision of my previous book, *Introduction to Nonlinear Oscillations* (Cambridge University Press, 1981). The earlier book was concerned with the application of various perturbation techniques to the construction of analytic approximations to the oscillatory solutions of second order, nonlinear differential equations. The present book continues this general theme. The standard perturbation methods of Lindstedt-Poincaré, Krylov-Bogoliubov-Mitropolsky, and multi-time expansions are given along with new chapters that discuss the method of harmonic balance and a general procedure for two coupled first order differential equations based on the Hopf bifurcation theorem and averaging.

As with the previous book, emphasis is placed on the genesis of the various techniques and how they can be applied to the differential equations that arise in mathematical models of phenomena from the sciences and engineering. The basic style of this book is heuristic rather than rigorous. However, a rather extensive set of references to the relevant research literature is given for each topic discussed. In addition, the General Bibliography provides a listing of books and monographs in the areas of differential equations, nonlinear oscillations, and nonlinear analysis. Each chapter contains a rather large number of "worked examples." They both illustrate the use of the techniques discussed in particular sections and provide insight and meaning to various concepts needed to understand the calculational procedures.

I thank my many colleagues around the world for their interest in both my previous book and my research over the past decade in nonlinear oscillations. This book is in large measure a response to their critiques, requests for reprints, and various types of collaborations. As always, I am particularly grateful to Annette Rohrs for typing the complete manuscript. Both she and my wife, Maria Mickens, provided valuable editorial assistance and encouragement. I also wish to thank Lori Darden who provided me with the superb figures used in this book. The work reported here was supported in part by grants from ARO, NASA, and NIH (MBRS).

Ronald E. Mickens
Atlanta, Georgia
August 1995

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