Photo by Hugh K. Robertson, Vancouver

George Bojadziev obtained his PhD in Mathematics at the Technical University of Sofia, Bulgaria. He was Professor at the same university until 1970. Since 1971 he has been with Simon Fraser University, British Columbia, Canada. His research activities in nonlinear phenomena, perturbation methods, stability, population dynamics, neural networks, intelligent control of robot manipulators, and fuzzy logic have resulted in more than 160 research publications (some of which he coauthored) and 4 books. Currently he is Professor Emeritus.

Maria Bojadziev obtained her Professional Degree in Mechanical Engineering at the Technical University of Sofia. She was Assistant Professor at the University for Chemical Engineering, Sofia. Since 1975 she has been with British Columbia Institute of Technology, British Columbia, Canada, as Mathematics Instructor. Her major research interests are nonlinear vibrations, dynamics and control of robot manipulators, and fuzzy logic control. She has authored or coauthored 30 papers and organized 3 workshops on fuzzy logic and intelligent control.

ADVANCES IN FUZZY SYSTEMS — APPLICATIONS AND THEORY

Honorary Editor: Lotfi A. Zadeh (*Univ. of California, Berkeley*)

Series Editors: Kaoru Hirota (*Tokyo Inst. of Tech.*),
George J. Klir (*SUNY at Binghamton*),
Elie Sanchez (*Neurinfo*),
Pei-Zhuang Wang (*Nat. Univ. of Singapore*),
Ronald R. Yager (*Iona College*)

Vol. 1: Between Mind and Computer: Fuzzy Science and Engineering
(Eds. P.-Z. Wang and K.-F. Loe)

Vol. 2: Industrial Applications of Fuzzy Technology in the World
(Eds. K. Hirota and M. Sugeno)

Vol. 3: Comparative Approaches to Medical Reasoning
(Eds. M. E. Cohen and D. L. Hudson)

Vol. 4: Fuzzy Logic and Soft Computing
(Eds. B. Bouchon-Meunier, R. R. Yager and L. A. Zadeh)

Vol. 5: Fuzzy Sets, Fuzzy Logic, Applications
(G. Bojadziev and M. Bojadziev)

Vol. 8: Foundations and Applications of Possibility Theory
(Eds. G. de Cooman, D. Ruan and E. E. Kerre)

Forthcoming volumes:

Vol. 6: Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers
by Lotfi A. Zadeh
(Eds. G. J. Klir and B. Yuan)

Vol. 7: Genetic Algorithms and Fuzzy Logic Systems: Soft Computing
Perspectives
(Eds. E. Sanchez, T. Shibata and L. A. Zadeh)

Advances in Fuzzy Systems — Applications and Theory Vol. 5

FUZZY SETS, FUZZY LOGIC, APPLICATIONS

George Bojadziev

Simon Fraser University

Maria Bojadziev

British Columbia Institute of Technology

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co Pte Ltd

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Bojadziev, George.

Fuzzy sets, fuzzy logic, applications / George Bojadziev, Maria Bojadziev.

p. cm. -- (Advances in fuzzy systems ; vol. 5)

Includes bibliographical references and index.

ISBN 9810223889 ISBN 9810226063 (pbk)

1. Automatic control. 2. Fuzzy sets. 3. Fuzzy logic.

I. Bojadziev, Maria. II. Title. III. Series.

TJ213.B59518 1995

629.8--dc20

95-45806

CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 1995 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, Massachusetts 01923, USA.

Printed in Singapore.

Dedicated to the 30th anniversary (1965–1995)
of the creation of Fuzzy Sets and Fuzzy Logic,
the Science bridging mathematical precision and
vagueness of common-sense reasoning.

This page is intentionally left blank

Contents

Preface	xi
1 Interval Arithmetic	1
1.1 Interval Numbers	1
1.2 Arithmetic Operations	4
1.3 Rules for Operations	7
1.4 Distance Between Intervals	10
1.5 Interval Operations in Z	11
1.6 Exercises	12
1.7 Notes	14
2 Multi-Level Interval Numbers	15
2.1 Two-Level Interval Numbers	15
2.2 Arithmetic Operations with Two-Level Intervals	16
2.3 More General Two-Level Intervals	20
2.4 Interval Numbers with n Levels	21
2.5 General n -Level Intervals	22
2.6 Infinite-Level Interval Numbers	23
2.7 Exercises	26
2.8 Notes	27
3 Fuzzy Numbers	29
3.1 Fuzzy Numbers with a Maximum	29
3.2 Triangular Fuzzy Numbers	35
3.3 Bell-Shaped Fuzzy Numbers	41
3.4 Fuzzy Numbers with a Flat	44
3.5 Trapezoidal Fuzzy Numbers	45
3.6 Piecewise-Quadratic Fuzzy Numbers with a Flat	48
3.7 Fuzzy Numbers Describing 'Large'	50

3.8	Fuzzy Numbers in the Set of Integers	56
3.9	Fuzzy Numbers of Dimension Two	59
3.10	Exercises	63
3.11	Notes	67
4	Arithmetic with Fuzzy Numbers	69
4.1	Introduction to Operations with Fuzzy Numbers	69
4.2	Addition of Fuzzy Numbers	70
4.3	Subtraction of Fuzzy Numbers	76
4.4	Multiplication of Fuzzy Numbers	78
4.5	Division of Fuzzy Numbers	84
4.6	Distance Between Triangular Fuzzy Numbers	85
4.7	Fuzzy Operations in the Set of Integers	87
4.8	Exercises	93
4.9	Notes	94
5	Classical Sets	95
5.1	Basic Definitions and Operations	95
5.2	Relations	99
5.3	Characteristic Function	104
5.4	Exercises	109
5.5	Notes	110
6	Fuzzy Sets	113
6.1	Definition of Fuzzy Sets	113
6.2	Fuzzy Sets and Fuzzy Numbers	121
6.3	Basic Operations on Fuzzy Sets	122
6.4	Properties of Fuzzy Sets	127
6.5	Algebraic Product and Sum of Fuzzy Sets	129
6.6	Power and Related Operations on Fuzzy Sets	131
6.7	The Extension Principle	133
6.8	Exercises	135
6.9	Notes	139
7	Fuzzy Relations	141
7.1	Definition of Fuzzy Relation	141
7.2	Basic Operations on Fuzzy Relations	145
7.3	Direct Product	148

7.4	Projections of a Fuzzy Relation	149
7.5	Max–Min and Min–Max Compositions	150
7.6	Basic Properties of Fuzzy Relations	153
7.7	Fuzzy Relations and Approximate Reasoning	154
7.8	Exercises	155
7.9	Notes	158
8	Classical and Many-Valued Logic	159
8.1	Basic Concepts of Classical Logic	159
8.2	Propositional and Predicate Calculus	164
8.3	Three-Valued Logic	170
8.4	Many-Valued Logic	172
8.5	Exercises	174
8.6	Notes	175
9	Fuzzy Logic	177
9.1	What is Fuzzy Logic?	177
9.2	Linguistic Variables	178
9.3	Linguistic Modifiers	180
9.4	Truth	183
9.5	Propositions of Fuzzy Logic	185
9.6	Composition Rules for Propositions	186
9.7	Quantification Rules	189
9.8	Qualification Rules	192
9.9	Semantic Equivalence	194
9.10	Semantic Entailment	195
9.11	Approximate Reasoning	198
9.12	Exercises	203
9.13	Notes	206
10	Decision Making and Applications	209
10.1	Decision Making	209
10.2	Fuzzy Delphi Method for Forecasting	214
10.3	Fuzzy Zero-Based Budgeting	219
10.4	Exercises	224
10.5	Notes	227

11 Fuzzy Logic Control and Applications	229
11.1 Introduction	229
11.2 Modeling the Control Parameters	231
11.3 If ... and ... Then Rules	233
11.4 Rule Evaluation	235
11.5 Conflict Resolution	237
11.6 Defuzzification	244
11.7 Washing Machine	246
11.8 Fuzzy Logic Control for a Predator–Prey System	252
11.9 Exercises	257
11.10 Notes	259
 Answers, Hints, Solutions to Selected Exercises	 261
 References	 271
 Index	 275

Preface

“Vagueness is no more to be done away with in the world of logic than friction in mechanics.”

Charles Peirce

Reality is more or less uncertain, vague, and ambiguous.

In the last three decades significant progress has been made in the development of fuzzy sets and fuzzy logic theory and their use in a large variety of applied topics in engineering and natural and socio-economic sciences involving uncertainty, vagueness, and ambiguity.

Fuzzy sets and fuzzy logic, founded in the mid sixties by Lotfi Zadeh, professor of electrical engineering and computer science at Berkeley, U.S.A, can be viewed as a broad conceptual framework enclosing the classical sets and logic.

The classical sets and mathematical logic divide the world of yes and no, white and black, true and false. Fuzzy sets and fuzzy logic deal with objects that are “matter-of-degree,” with all possible grades of truth between yes and no, and the shades of grey between white and black.

Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature, and humanity; and facilitators for common-sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution.

Fuzzy logic is attracting a great deal of attention in the business and industrial world as well as among the general public.

The Japanese scientists and engineers were quick to recognize the enormous potential of fuzzy logic and have successfully implemented

fuzzy logic control in numerous commercial products such as vacuum cleaners, washing machines, rice cookers, photocopiers, showers, etc. resulting in energy efficiency and increased convenience for the consumer. The Japanese city of Sendai has been using subway trains controlled by fuzzy logic since 1986.

Fuzzy sets and fuzzy logic have been applied virtually in all branches of science, engineering, and socio-economic sciences.

The text of this book aims to help university and college students, as well as scientists and technologists, to learn the basics of fuzzy logic and enable them to make relevant applications.

Fuzzy logic and fuzzy set theory are relatively new fields. The main efforts of those actively working on these have been concentrated either on theoretical and applied research published in journals and proceedings or the development of software. Also there are monographs and advanced texts on fuzzy sets and fuzzy logic covering extensively the theory or the applications, or both. However it is difficult to find an introductory text. The aim of this book is to fill this gap and bring the mathematics of fuzzy sets and fuzzy logic into the educational system in a systematic way. Both students and teachers will be able to benefit from the book which can be used as a major resource for learning theory, solving problems, and initiating research projects.

The text is intended for an introductory course in fuzzy sets and fuzzy logic.

No previous knowledge of fuzzy sets and fuzzy logic is required for the understanding of the material. The reader is assumed to be familiar with high school algebra and the concept of function. The emphasis is on the interpretation of the mathematics rather than on details of the derivations of theorems.

The book can be used by students in engineering, computing, sciences, economics, and arts in universities and colleges.

The authors believe that the text will significantly contribute to paving the way for regular teachings of fuzzy sets and fuzzy logic in the educational institutions in many countries and arouse interest in a large number of scientists without prior knowledge in fuzzy theory.

The text is based on class notes prepared for a course delivered at British Columbia Institute of Technology, Canada, in 1993 and 1994. It can be characterized as an applied mathematics book with interdisci-

plinary nature due to the emphasis on applications in engineering and socio-economic sciences.

The book consists of eleven chapters subdivided into various sections. Each chapter closes with short notes concerning bibliography and miscellaneous pieces of additional information: historical, philosophical, mathematical, etc.

There are 116 illustrative examples in the book and exercises follow after each chapter; also answers, hints, and solutions are given to selected exercises. The book is supplied with references and a subject index.

Usually books on this subject start with fuzzy set theory and in particular consider briefly (or not at all) fuzzy numbers.

We begin differently, with interval numbers and computations with interval numbers (Chapter 1). They are often used by numerical analysts, computer scientists, and those performing computations using imprecise data. Multi-level interval numbers, an extension of interval numbers, are presented in Chapter 2. They are used to introduce in a natural way the concept of fuzzy numbers in Chapter 3 and arithmetic operations with fuzzy numbers (Chapter 4). This inductive approach is novel. Fuzzy numbers have a greater expressive power than interval numbers due to the capability of gradation on various levels. They are well suited to describe linguistic variables (for instance small, large, age, tall, speed, power, force, etc.).

A brief review of classical set theory is presented in Chapter 5. The selection of the topics assists the introduction of the concept fuzzy set in Chapter 6. It is shown that fuzzy numbers can be obtained as a special case of fuzzy sets. A fuzzy set defines the degree to which an element belongs to the set. Fuzzy sets, similarly to fuzzy numbers, are used to describe linguistic variables. Fuzzy sets play an important role in both applications and theoretical studies involving various types of fuzziness. They are instrumental for the development of fuzzy logic.

Chapter 7 covers fuzzy relations together with their basic operations. Fuzzy relations are introduced as a generalization of fuzzy sets and ordinary relations. They provide the foundation for approximate reasoning and fuzzy logic control.

A short description of classical and many-valued logic is presented in Chapter 8. Classical (two-valued) logic deals with proposition that

are either true or false. In many-valued logic, a generalization of the classical logic, the propositions have more than two truth values.

Fuzzy logic (Chapter 9) is an extension of many-valued logic in the sense of incorporating fuzzy sets and fuzzy relations as tools into the system of many-valued logic. Fuzzy logic facilitates approximate reasoning with imprecise propositions dealing with natural language.

Chapter 10 covers briefly decision making, a process of problem solving involving pursuing of goals under constraints. Usually a decision-making process takes place in a fuzzy environment, hence it is appropriate to model goals and constraints by fuzzy sets. Applications in management and budgeting are also presented.

Chapter 11 is devoted to fuzzy logic control. Fuzzy sets and fuzzy logic applied to control problems form a field of knowledge called fuzzy logic control. It is shown how control problems can be solved by rules based on approximate reasoning without using conventional mathematical models. Such models for many complex processes might be difficult or impossible to construct. The methodology, illustrated on the inverted pendulum and washing machine, can easily be applied to numerous control problems in industry. The section on fuzzy logic control for a predator-prey system contains original results.

Acknowledgments

We would like to express our gratitude to Prof. Lotfi Zadeh for his encouragement and influence. His contributions to and achievements in science and engineering, together with the contributions of many followers of his ideas, established the *Science of Fuzzy Sets and Fuzzy Logic*. Those achievements provided the basis for this textbook.

We owe a debt of thanks to the authors whose books or articles are listed in the references. Many of the ideas and examples in this book stemmed from those sources.

We also thank Q. Joy Wang and Hanlong Yang for typing with skill and patience the manuscript with its many figures and tables.

Finally, we are grateful to World Scientific Publishing Company for bringing out this book and in particular to Yew Kee Chiang for his skillful and excellent editorial work.

Vancouver, Canada
September 1995

George Bojadziev
Maria Bojadziev