Advanced Calculus

An Introduction to
Mathematical Analysis

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An Introduction to
Mathematical Analysis

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PREFACE

The present book is, as its title indicates, a presentation of some fundamental ideas related to the elementary real analysis.

For a better understanding of the material given here, a course in basic differential and integral calculus can be a good preliminary preparation.

The emphasis in our present text lies on the so-called rigorous method where everything (well, *almost* everything) is given a clear definition, a detailed statement, a complete, logically coherent proof.

The book starts with an exposition of the theory of real numbers; it is this theory which is taken as a basis for the concepts developed afterwards. We present real numbers as equivalence classes of Cauchy sequences of rational numbers.

This is different from what can be found in most recent books, where one prefers the axiomatic way or (sometimes) the method of “Dedekind cuts.” We consider the method of Cauchy sequences a very clear manner of introducing general, real numbers; furthermore, it has the advantage of being applicable in other situations, for instance in the theory of “metric spaces.” Afterwards we give the usual, modern presentation, of such topics as: sequences of real numbers, infinite numerical series, continuous functions, derivatives and integration theory.

There are also two chapters of a peculiar type: the first one concerns convex functions, a class of functions which appear useful in many applications of calculus; the second one, about metric spaces, reflects a recent tendency to start presenting “topological” ideas from the very beginning of the undergraduate mathematical life.

The attentive reader will note probably the absence of most, “well-known” elementary functions, like $\sin x$, $\cos x$, e^x , $\log x$, from the book. They are

never used here; their rigorous presentation would take many more pages and the student learns about them in any case in other places.

At the end of each Chapter we added some quite simple exercises, which, if solved, would help for a higher understanding of the subject matter previously treated.

We terminate the book with an Appendix concerning general concepts of logic and set theory which are used in the text. There is also an Index of Notations, an Index of Subjects and a short list of bibliographical references. The student should find them useful.

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