FINITE TEMPERATURE FIELD THEORY

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To
Kanjapa
and
Krushna

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Introduction

In the past several years, there have been a lot of interest in various phenomena, in quantum field theories, at finite temperature. The formalism, for such field theoretic investigations, of course, dates back to Matsubara. However, over the years, there have been alternate methods of doing calculations in field theories at finite temperature which shed light on the structure of theories at finite temperature. And, as more and more studies are carried out in these formalisms, we become more and more aware of the subtleties at finite temperature and we understand the structure of these theories much better.

While many people are quite familiar with the Matsubara formalism or the imaginary time formalism, the alternate, real time formalisms are not as widely appreciated. However, the analysis of many phenomena of current interest need such alternate formalisms. Keeping this in mind, I have tried to describe all the three formalisms – imaginary time formalism, closed time path formalism and thermofield dynamics – right in the beginning chapters of this book. There are numerous examples that are worked out to give a sense of which formalism may be better suited for a given kind of problem. It is, of course, left completely to the reader to choose from among these formalisms, to carry out any calculation.

I also develop, in detail, various other concepts that are not as widely stressed in the literature. For example, the study of gauge theories at finite temperature, the cutting rules at finite temperature, the gauge dependence of the effective potential are just a few topics that are discussed at great lengths. Most of these topics are basic field theory questions and, therefore, to keep the material self-complete, many of the zero temperature topics on field theory are also worked out in detail.

The idea here is to bring out, from these finite temperature calculations, the similarities with and differences from what we know at zero temperature. Although, I had originally planned to include a discussion of the hard thermal loops in this book, neither space nor time allowed this. Moreover, I recognize that there exist, by now, several excellent sources of discussion of the hard thermal loops in the literature. The references, at the end of each chapter, only represent those which I have found useful in presenting the material. There are many more papers on the various topics being discussed and I apologize for not being able to include an exhaustive list of references.

The material in the first few chapters of this book were presented as a series of lectures at the Mehta Research Institute, Allahabad and I would like to thank Prof. H.S. Mani for his hospitality. I have benefited and learned a lot from all of my collaborators on the subject – K.S. Babu, P.F. Bedaque, M. Hott, A. Karev, S. Naik, S. Panda, P. Panigrahi and S. Vokos – and I would like to thank them all. I also acknowledge numerous suggestions (and help in simplifying many difficult derivations) from Drs. P.F. Bedaque and S. Pernice as well as from Profs. S. Okubo and H.A. Weldon. I would also like to thank M. Begel and T. Guptill for much needed assistance on computer related matters. All the figures in this book were drawn using PSTricks.

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Rochester

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