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Preface

This book is an extended version of my book *Stability theory of elastic rods*, written in Serbian, and published by Faculty of Technical Sciences in Novi Sad 1987. It grew out of the lectures that I was giving to the fourth year engineering students, specializing in applied mechanics at the University of Novi Sad. It treats two specific parts of the stability theory of elastic bodies. First, it is concerned with the stability of *equilibrium* states and second, it treats elastic *rods* only. The mathematical methods presented here could, however, give an insight into the stability analysis of a given state, not necessarily equilibrium, of an elastic body to be investigated. For the case of relative equilibrium, that is, equilibrium with respect to a moving coordinate system the methods presented here, apply directly. Also stability analysis of elastic rods shows, in principle, how to proceed in the case of two-dimensional (plates and shells) and three-dimensional bodies. I hope that the present text will be read in conjunction with the classics in the field such as: Timoshenko, S.P. and Gere, J.M. *Theory of Elastic Stability*, Volmir, A.S. *Stability of Deformable Systems*, Knops, R.J. and Wilkes, E.W. article *Theory of Elastic Stability in Handbuch der Physik* Vol. 3 and Leipholz, H. *Stability of elastic Systems*.

The book is addressed to senior undergraduate students with emphasis on mechanics and (or) applied mathematics and graduate engineering students. It is divided in five Chapters.

Chapter I is conceived as an introduction where general notions of rod, rod axis and rod cross-sections are presented. Also, we discuss possible rod models for stability analysis and give basic ideas about Euler, energy and dynamic method of stability analysis. The notions of structural stability and practical stability are introduced as well.

Chapter 2 deals with basic equations of the elastic rod theory. Plane deformations are considered first. The classic Bernoulli–Euler model follows as a special case of various generalized models. Special attention is given to models that take shear stress effects into account and to classification of such models. We distinguish three approaches: Engesser’s, Haringx’s and Timoshenko’s. In treating spatial deformations of elastic rods three different choices of Euler type angles describing orientation of a cross-section with respect to a fixed Cartesian coordinate system are introduced. This chapter also gives solution of equilibrium equations for certain types of loads and boundary conditions. Among the classic solutions, we analyze an inverse problem (the shape of the piston ring) and a curiosity called negativism. An important characteristics of this Chapter is that only linear (in selected strain measures) constitutive equations are used.

Chapter 3 begins with stability analysis. First the Euler method of adjacent equilibrium configuration is treated. It is shown that the question of instability or stability of a certain equilibrium configuration is mathematically reduced to the problem of determining whether certain non-linear operator does or does not have a bifurcation point. The Liapunov–Schmidt reduction is described as one possible method of determining bifurcation point. The method is illustrated by several examples. Some are classic ones and some are newer. In presenting the classic examples, we tried to include various generalizations and extensions that appear in literature. This is specially true for Sections 3.3, 3.10, 3.13, 3.18 and 3.19.

Chapter 4 deals with the energy method for stability analysis. The energy method is taken as an independent method of stability analysis and its relation to Euler method is examined. Then, a procedure is formulated to determine stability boundary according to the energy method. This procedure is fully escorted on a model example in Section 4.2. Later in Chapter 4 a selected number of examples is treated. Some of these examples were previously treated by Euler method in Chapter 3. In Section 4.9 several variation principles and a priori estimates of maximal deflection are presented. It is shown then how those estimates could be used to estimate the critical load for the case of rods with variable cross-sections.

Chapter 5 contains the dynamic method of stability analysis. Both first and second Liapunov method are presented together with Liapunov stability theorems. Connection with the energy method is discussed. The method is then applied to several examples. This chapter also contains generaliza-

tion of some classic results. For example, the influence of shear, compressibility and pulsating force is examined on the case of Beck's and Pflüger's rods. This Chapter closes with the generalization of minimal length necessary for buckling formula obtained by the dynamic method.

The References, given at the end, are by no means complete. Instead, our intention was to cite only works that are relevant for the examples treated.

Finally, I would like to express my gratitude to Professors B.D. Vujanovic, University of Novi Sad and I. Müller, Technische Universität Berlin for encouraging me to write this book. With my colleges B. Baclic, Dj.S. Djukic and L. Cveticanin I had numerous discussions about the content. Professor Baclic helped me to type, review and edit the manuscript. This help was essential. Professor A. Guran, University of Southern California, helped me to improve the final version of the text and suggested extension of some sections. It is also my pleasure to acknowledge the financial support of Alexander von Humboldt-Stiftung of the Federal Republic of Germany. During my stay in Berlin, financed by Alexander von Humboldt-Stiftung, some parts of this book were written.

The help that I received from my daughters Jelena and Milica is hard to describe. But it was crucial. I therefore dedicate this book to them.

Novi Sad, 1996

T.A.

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