TOPICS ON STABILITY AND PERIODICITY IN ABSTRACT DIFFERENTIAL EQUATIONS

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Preface

Asymptotic behavior of evolution equations is a well-studied area in the theory of abstract differential equations with various methods of studies. It is natural to use the well-known ideas and techniques in the finite dimensional case as much as possible to deal with the problems in the infinite dimensional case. Having this in mind, in this book we will make an attempt to gather systematically certain recent results on several central topics of the asymptotic behavior of differential equations in Banach spaces. We will discuss the conditions for the stability, dichotomy and harmonic oscillation of solutions of evolution equations. The results and methods of approach will be presented in a manner that allows the reader, who is familiar with the techniques in the finite dimensional case, to easily understand them. Some parts of the book are actually lecture notes we have taught to graduate students over the past years.

We outline briefly the contents of our book. In Chapter 1 we recall several basic facts from semigroup theory, spectral theory of functions that will be used throughout the book. Chapter 2 is devoted to some classical topics including stability and dichotomy of linear homogeneous equations. In Chapter 3 we present some new methods of studying the harmonic oscillation in inhomogeneous linear equations. Chapter 4 is devoted to the topic of almost automorphy of solutions, that has recently regained interest in the mathematical literature. Existence of almost automorphic solutions to some linear and semilinear abstract differential equations is studied. We discuss the Massera type conditions for the existence of periodic solutions to periodic nonlinear equations in Chapter 5. At the end of each chapter we give a guide for further reading and comments on the results as well as the methods of study discussed in the chapter. We finally collect some of the required tools from functional analysis and operator theory in the

appendices.

We wish to thank our colleagues and students for their encouragement and patience during the last years.

James H. Liu, Gaston M. N'guerekata, Nguyen Van Minh

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