Analysis — and — Topology

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Analysis — and — Topology

**A volume dedicated to
the memory of S. Stoilow**

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Simion Stoilow
(1887–1961)

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FOREWORD

This volume is dedicated to the memory of the great Romanian mathematician Simion Stoilow (1887–1961), the founder of the topological theory of analytic functions, which is the reason we give it the title *Analysis and Topology*. It brings together selected articles written by distinguished scientists whose research field is related to Stoilow's work.

Simion Stoilow made important contributions to various fields of mathematical analysis and topology, but his main achievement lies in his in-depth comprehension of the phenomenon of analyticity by means of topology: the solution to Brouwer's problem of characterizing analytic functions topologically by introducing the concept of interior transformation, the methods created for the topological study of functions (e.g. the path lifting, the normal domains), the definition of the covering Riemann surface and generally of the covering space, the introduction of the Kerékjártó–Stoilow boundary (which became a fundamental instrument in classifying abstract Riemann surfaces in the H. Weyl and T. Radó sense), the programme of classifying Riemann coverings (wherein he defined and thoroughly studied the classes of Iversen and of normally exhaustible coverings), as well as the Stoilow principle. Such has been Stoilow's contribution which has inspired and still inspires numerous research mathematicians worldwide.

In this volume, dedicated to the memory of the famous mathematician on the 110 year anniversary of his birthday, we have attempted to reflect a good part of Stoilow's contribution. The scope covered is quite varied: Riemann surfaces, Teichmüller theory, Kleinian groups, automorphic forms, the Riemann ζ -function, modulus and capacity, null-sets, potential theory,

conformal geometry, quasiconformality, complex differential and partial differential equations, variational inequalities and optimization, dynamical systems and functional equations. This shows once again how successful the synthesis of analysis and topology initiated by Stoilow can be.

The volume starts with a brief survey on his work and its influence on several mathematical fields, accompanied by an invaluable document “*Exposé sommaire de mon travail de recherche*”, written by S. Stoilow himself at the request of the French Academy during the last months of his life. We think that this seminal work captures the deep significance of Stoilow’s mathematical creation. We translate and publish it here in English together with a list of titles of his mathematical works.

The volume also includes several articles on Stoilow’s work as well as his role in creating the present Romanian school of mathematics; specifically, in creating the school of complex analysis, in organizing and directing the Institute of Mathematics of the Romanian Academy, both as President of the Mathematical and Physical Sciences Division of this Academy and as Director of its Mathematical Institute.

In addition to the above, Simion Stoilow had an impressive philosophic, literary and artistic culture. Whoever met him was struck by his kindness, his sense of justice, his promotion of right causes, his noble behavior under adversity. It was this side of his character, coupled with his mathematical genius, that helped establish the friendship with the famous Finnish mathematician Rolf Nevanlinna, the well-known Finnish–Romanian collaboration which indeed proved to be influential in international mathematics.

It is a great pleasure for us to express our gratitude to all the scientists from approximately 15 countries, who contributed to this volume. We truly thank all the other collaborators, especially Doina Irina Simion, for her translation of Stoilow’s “*Exposé sommaire de mon travail de recherche*” and other papers. We also wish to acknowledge the superb assistance of World Scientific in this joint effort.

Cabiria Andreian Cazacu
Olli E. Lehto
Themistocles M. Rassias

January 1998

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