Quasilinear Hyperbolic Systems and Dissipative Mechanisms

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Preface

The study of quasilinear hyperbolic systems that result from the balance laws of continuum physics was initiated more than a century ago, and considerable progress in this field has been made although many fundamental problems in the analytical theory remain unsolved.

The aim of this book is to introduce recent developments on the subject of quasilinear hyperbolic systems with dissipation, particularly systems of hyperbolic conservation laws with either lower order dissipation such as frictional damping and relaxation, or higher order dissipation such as viscosity and heat diffusion. Other kinds of dissipative mechanisms, such as those induced by “memory” or boundary conditions, will not be considered.

The purpose is, on the one hand, to understand the influence of dissipation on the qualitative behavior of solutions, such as the nonlinear diffusive phenomena, phase transitions etc. and on the other hand to employ dissipation as a device for approximating solutions of hyperbolic systems by passing to the zero viscosity or zero relaxation limit.

The first two chapters are devoted to a research program which connects the area of equations with nonlinear diffusion with the area of quasilinear hyperbolic systems. Nonlinear diffusion problems arising in various applied sciences have been widely investigated in recent years and connections with well understood, more “abstract” subjects in the theory of nonlinear parabolic equations have been uncovered. The class of systems with damping mechanisms has attracted particular attention because they model a number of problems such as the flow of an elastic fluid through a porous medium or in a pipeline, the oscillation of an elastic string immersed in a fluid, etc. In Chapter 1, we show that the large-time behavior of classical solutions of systems of hyperbolic conservation laws with frictional damping is related to the large time behavior of solutions of systems with nonlinear diffusion. Chapter 2 is devoted to weak solutions for such systems, either in the class of piece-wise continuous and piece-wise smooth functions or in the class of BV functions.

Many physical situations require consideration of the effects of relaxation. For instance, in the kinetic theory relaxation is related to the mean free path and in viscoelasticity it is induced by memory. In gas dynamics, the relaxation phenomenon occurs when the gas is in thermo-non-equilibrium. It also occurs

in chromatography, river flow, traffic flow, etc. Therefore, a natural approach is to approximate hyperbolic systems of conservation laws by systems that involve relaxation. We investigate the topics of stability of elementary waves and singular limits of zero relaxation time in Chapter 3.

Chapter 4 investigates the influence of higher order dissipation on the qualitative behavior of solutions, such as viscosity and heat diffusion. In general, dissipative mechanisms may affect the asymptotic behavior as well as the smoothness of solutions. The stabilizing effect on the solutions will be investigated in Section 4.1. It is shown, for instance, that the combined dissipation of viscosity and heat diffusion will preserve the smoothness of the initial data without restrictions on smallness. Another aim of this Chapter is to understand the influence of dissipation on the large-time behavior of solutions. Various phenomena, including phase transition phenomena, are exhibited on the large-time behavior of solutions. This is discussed in Section 4.2. The last section, Section 4.3, is devoted to the study of behavior of solutions depending on different boundary conditions. To obtain the a priori estimates guaranteeing either the global existence or the expected large-time behavior of solutions, careful analysis based on energy approach has been carried out.

The last Chapter, Chapter 5, considers a viscous system of conservation laws. The first topic is to study the relationship between the solutions of the viscous system considered and the distributional solutions of the corresponding hyperbolic system of conservation laws without viscosity, especially the problem of vanishing viscosity which is discussed in Section 5.1. Another closely related topic is to study the similarities and differences of the large-time behavior of solutions for these two systems, particularly, to compare the large-time behavior of solutions for the viscous system to the solutions of the Riemann problem for the corresponding system of quasilinear hyperbolic conservation laws. This problem of nonlinear stability of waves will be discussed in Section 5.2. Section 5.3 investigates the above two topics with regards to multi-space dimensional systems.

The bibliography contained in this book provides only a sample of the relevant literature and is far from comprehensive.

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Contents

Preface	v
1 Frictional damping: Globally defined classical solutions and their nonlinear diffusive phenomena	1
1.1 The system of compressible isentropic flow with damping	3
1.2 The global existence of smooth solutions for the system of compressible adiabatic flow with damping	9
1.3 The nonlinear diffusive phenomena of smooth solutions for the system of compressible adiabatic flow with damping	15
2 Frictional damping: Globally defined weak solutions and the nonlinear diffusive phenomena	35
2.1 Riemann problem and perturbed Riemann problem	36
2.2 Nonlinear diffusive phenomena for entropy weak solution	51
2.3 Globally defined admissible BV solutions	64
3 Relaxation	67
3.1 The stability of shock profiles for a rate-type viscoelastic system with relaxation	67
3.2 The stability of planar rarefaction waves and shock fronts for the relaxation approximations of conservation laws in several dimensions	89
3.3 The zero relaxation limit	102
4 The influence of dissipation mechanism on the qualitative behavior of solutions	111
4.1 The influence of dissipation mechanism on the smoothness of solutions	114
4.2 The influence of dissipation mechanism on the large-time behavior of solutions	128
4.3 Various large-time behavior corresponding to different boundary conditions	146

5	Vanishing viscosity and nonlinear stability of waves	165
5.1	Vanishing viscosity – the zero dissipation limit	166
5.2	Nonlinear stability of waves	178
5.3	The multidimensional case	193
	Bibliography	207
	Index	222