QUANTA, LOGIC AND SPACETIME

**VARIATIONS ON
FINKELSTEIN'S QUANTUM RELATIVITY**

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FINKELSTEIN'S QUANTUM RELATIVITY**

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Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN 981-02-3255-1

Printed in Singapore.

For Jenny

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Preface

These notes have their origin in a reading (in 1991) of David Finkelstein's "Space-Time Code" papers (Finkelstein 1969, 1972a, b, 1974; Finkelstein, Frye and Susskind 1974), their subsequent refinements and collateral descendants.

In particular, the extraordinary first paper in the Space-Time Code series (Finkelstein 1969) rather shockingly derives the basic elements of special relativity from quantum logical principles. Roughly speaking, it is shown that if the pointlike objects which are presumed to underlie the plenum are treated according to the rules of *quantum* logic—rather than classical (set theoretic) logic—then the basic elements of relativity emerge spontaneously. Quantum theory, far from being in conflict with relativity, is shown actually to underlie it.

This compelling insight has led Finkelstein and coworkers on a continuing quest for a theory which fuses quantum theory with relativity at a deep level. An account of this work, as well as full references to related literature, is to be found in Finkelstein's monograph "Quantum Relativity" (Springer 1996) which, as our primary source, will be abbreviated to QR in the sequel.

It is often difficult, if not impossible (even for the protagonists), to pinpoint the precursors of those theoretical works which effect a paradigm shift. This is a task best left to professional historians. Nevertheless, certain influences should be mentioned here. Of the two major conceptual strands that contribute to the fabric of the Space-Time Code papers, namely space-time *causal structure* and *quantum logic*, the first was very much on the minds of other relativists in the 1960s, the second less so, if at all. Combinatorial or discrete models of spacetime had also been toyed with since at least the 1940s. A mathematically sophisticated descendant of these was produced by R. Penrose in the late 1960s: namely, the *spin net*. Among the innovations here was the weaving together of *spinors* and combinatorial structure to produce a spacetime antecedent, and Finkelstein acknowledges the strong influence of this work on his own ideas. (Penrose's spin nets have recently staged a come-

back in the theory of loop representations of gauge theories. See for instance Baez 1996.) On the other hand, some quantum physicists, perhaps working closer to the epistemological edge, had for decades been deeply suspicious of the classical spacetime with which necessity had saddled them. Occasional protests notwithstanding, this union of convenience would not be seriously threatened until extremely high energies (and/or the incorporation of gravity) were contemplated. Nevertheless, at least one visionary had understood early on (by the 1940s) the fundamental rôle that quantum logic should play at this level, and, among other remarkable insights, had explicitly suggested a connection between representations of $SU(2)$ and possible quantum versions of the “binary alternative”. This was C. F. von Weizsäcker, another acknowledged influence on Finkelstein’s synthesis. (See QR for references.)

Starting from a small subset of notions culled primarily from QR I aim to show in these notes that it is possible in principle to reconstruct *ab initio* a fair slice of fundamental physics, as seen by a (relatively!) macroscopic observer. Although I set out explicitly from (a subset of) Finkelstein’s foundations, ignorance of physics at the scale considered often forces me to supply formal or mathematical arguments: nevertheless, such results as are obtained, are, I think, in striking agreement with the Standard Model, at least for a massless world. Specifically, Finkelstein’s suggestion that some sort of quantum set-like structure, dubbed a “quantum net”, should underlie our macroscopic perception of spacetime, is taken up and used—in conjunction with a fairly lengthily articulated correspondence principle—to derive Lagrangian densities which take the form of power series in a “net constant” (having the dimension of length) whose lowest order terms reproduce the standard forms for massless Dirac fermions, General Relativity, and Yang–Mills fields for the gauge groups $U(1) \times SU(2)_L$, and $SU(3)$. These Lagrangians emerge replete with (Feynman) gauge-fixing terms; the associated ghost fields are obtained at little extra cost. A comparison with the standard Lagrangians at the “grand-unified” scale then yields an estimate for the net constant of about $19l_P$, where l_P ($\approx 1.6 \times 10^{-33}$ cm.) denotes the Planck length.

Neither a candidate for a Higgs-like field nor a mechanism to drive generational differences are immediately apparent, and many quantum numbers seem to be missing. In addition, the gravitational term is accompanied by a computationally intractable extra piece, so it is possible that we have avoided Pauli’s Curse (of not even being wrong). On the other hand, a fairly conspicuous candidate for a chiral symmetry breaking mechanism for the $SU(2)$ component does present itself. Problems with the theory are discussed in Chapter

9.

Our methods are largely algebraic, formal, and entirely elementary, so it is not surprising that the final product is incomplete. In defense of these methods one may argue that intuition—both physical and geometrical—is likely to be unreliable at what turns out to be the Planck scale: some level of formality seems to be mandatory. Indeed, Finkelstein has advised the voyager into these submicroscopic regions to travel light. Ignorance has made this initial unburdening easier for me, while increasing the dangers of the journey. Among the bulkier pieces I have carried with me I should mention:

- the fundamental classical notion of *binary alternative*, which has been retained in quantized form as the foundation of the whole structure;
- a fairly liberal use and interpretation of the algebraic notion of *derivation*, and what amounts to an associated Schwinger-like action principle, though expressed in path integral form;
- certain macroscopic geometrical notions such as “infinitesimal” parallel transport (which is, however, globalized in a rather formal way in Chapter 5), and a little Lie theory, used to motivate certain constructions in Chapters 3 and 8.

(Other macroscopic or continuum-based notions have undoubtedly stowed away, despite earnest efforts to evict them.)

On the other hand, among the baggage to be left behind at the outset I have tried to include:

- prejudices concerning the invariances, gauge or otherwise, of the expressions which will emerge as Lagrangian densities;
- prejudices concerning the rôle, or even the existence, of a spacetime metric.

The first of these is required to avoid an obvious circularity—at the cost of having to derive BRST invariant expressions from first principles. The issues concerning the spacetime metric are perhaps more inflammatory. Briefly stated, macroscopic experimenters become aware of the existence and behavior

of the metric in the continuum entirely as a result of the passage through it of physical probes, such as photons. Since I aim to deduce the existence and behavior of such probes from prior structures (in the form of their Lagrangian densities), I cannot make *a priori* assumptions concerning even the *existence* of the metric, let alone the nature of its entry into the Lagrangians. Instead, I must wait patiently for the emergent dynamics to reveal their metrical entanglements.

The impulse behind this entire enterprise was the naïve question of a mathematician upon contemplating the arcana of modern physics: namely, where do the Lagrangians come from? The form of these expressions is often argued on the basis of assumed invariances with respect to presumed group actions, and/or renormalizability requirements—a multiple begging of the question. (In the case of arguments from gauge invariance one soon receives the heartbreaking news that the gauge must be fixed, and, a little later, that ghosts must be invoked.) These notes represent my attempt to answer this question using the only tools I could find that seemed to combine profundity with simplicity in workable measures: namely, those provided by the work of Finkelstein *et al.*, and summarized in QR. As remarked above, I have chosen only a small subset of these, and have wielded them differently. The consistency of the results in the face of possible manhandling by a non-physicist surely points to the correctness and robustness of Finkelstein's underlying conceptions. If readers are encouraged to strike out on their own path through QR, the main purpose of these notes will have been amply served.

I have not presumed in these notes to push the results towards anything resembling “quantum gravity”, being content with the degree of immodesty already achieved in trying to attain classical gravity, *inter alia*, at low energies. Since we start with what amounts to an assumed vacuum which fixes at four the dimension of the emerging manifold, it seems unlikely that much accommodation with any version of string theory (requiring high dimensional manifolds at the outset) can be expected, without radically altering the foundations. On the other hand, it is perhaps worth mentioning that the loop states of recent canonical gravity theory may be realized and interpreted within the current language (Chapter 5).

It is obvious that I owe a special debt of gratitude to David Finkelstein, not only for having inspired the vacuum in the first place, but also for his patience in attempting to share his deep insights with a tyro, and his unstinting hospitality while doing so. My thanks to him also for cogent criticism of earlier versions, and to him and Springer Verlag for permission to quote certain

passages from QR.

My outlook on these and many other matters has been broadened and deepened through fortunate encounters with many remarkable individuals, whose wisdom and generosity it is a pleasure to acknowledge. These include Michael Atiyah, Raymond Balbes, W. Henson Graves, Karl Heinrich Hofmann, Theodore Jacobson, John Madore, Anastasios Mallios, Wolfgang Mantke, Christopher J. Mulvey, Ronald Munson, Prabhakar Rao, Ioannis Raptis, Kenneth A. Ross, Subramanian Sankaran, Dana Scott, Rafael Sorkin, Moss Sweedler, Zhong Tang, Morwen Thistlethwaite, Alexander Vlasov, Grant Welland, Josef Wichmann, and the late Eugene P. Wigner. (None of these can be held responsible for misunderstandings, misconstruals and other errors.) My grateful thanks to all of them, to an anonymous referee for words of encouragement, to Dr. Richard Lim of World Scientific, whose kindly interest persuaded me to commit this material to paper, and to him and Mr. H. T. Leong, also of World Scientific, for generous typographical indulgences.

The work was supported, and speeded on its way, by two liberating grants: one, from the University of Missouri Research Board, and another from the Division of Academic Affairs of the University of Missouri–St. Louis. Special and profuse thanks are owed, in connection with these grants and manifold other matters, to Lola Hohlt, Deloris Licklider and Kimberly Taylor.

None of these pages would have seen the light without the expert computer assistance of Mike Schulte.

Finally, but primarily, I thank my wife Jenny, whose love and patience defy the understanding of all who know me.

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