Theory of the Navier-Stokes Equations

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Theory of the Navier-Stokes Equations

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This Volume is dedicated
to the memory of

Professor Anatoli P. Oskolkov
(†1995)

and

Professor Valeriy Y. Rivkind
(†1996)

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Introduction

The articles in this volume constitute the proceedings of the third international conference held at Oberwolfach under the title "The Navier-Stokes Equations: Theory and Numerical Methods". The organizers were John Heywood (Vancouver), Kyuya Masuda (Tokyo), Reimund Rautmann (Paderborn), and Vsevolod A. Solonnikov (St. Petersburg).

These articles make important contributions to a wide variety of topics in the Navier-Stokes theory: general boundary conditions, flow exterior to an obstacle, conical boundary points, the controllability of solutions, compressible flow, non-Newtonian flow, magneto-hydrodynamics, thermal convection, the interaction of fluids with elastic solids, the regularity of solutions, and Rothe's method of approximation.

The article by Gerd Grubb (Copenhagen) offers a general classification of boundary conditions for the Navier-Stokes equations, and a unified treatment of the corresponding initial boundary value problems by the reduction of degenerate parabolic problems to pseudodifferential problems, and the subsequent application of a general theory for these pseudodifferential problems.

The article by Reinhard Farwig (Darmstadt) and Hermann Sohr (Paderborn) gives a new treatment of the problem of steady flow past an obstacle, by a method of weighted L_p -estimates that avoids explicit integral representations. In the three-dimensional case, they provide new proofs and extensions of the famous theory of "physically reasonable solutions" of Finn, Babenko, and Galdi. The article by Hans-Christoph Grunau (Bayreuth) begins the development of an L_p -approach to the stability of such physically reasonable solutions. The article by Hideo Kozono (Nagoya) and Masao Yamazaki (Tokyo) addresses the particular difficulties that are encountered in treating flow in exterior domains when the prescribed limit at infinity is zero.

Control problems are pioneered in an article by Andrei V. Fursikov (Moscow), where the controllability of three-dimensional flow by Dirichlet boundary conditions is investigated. For arbitrarily given initial values, it is proven that there exist boundary values which will bring the solution to rest in finite time. In another unusual article, Michael Ruzicka (Bonn) and Jens Frehse (Bonn) investigate the regularity of steady solutions in dimensions 5 and greater, motivated by a number of surprising analogies with the famous global existence problem for three-dimensional nonstationary flow.

Two of the articles presented here concern the extension of classical Navier-Stokes theory to fluids with more general constitutive laws. Antonin Novotny (Toulon) contributes to the global existence theory for steady solutions of the compressible isentropic Navier-Stokes equations, using a method of supersonic decomposition. Mariarosaria Padula (Ferrara) and Adelia Sequeira (Lisbon)

prove the well-posedness of the boundary value problem of a vector transport equation that arises in the study of non-Newtonian fluids.

Three of the articles concern the coupling of the Navier-Stokes equations with other physical effects. Martin Rumpf (Freiburg) has analyzed the interaction between a thick elastic obstacle and a viscous fluid. Yoshiyuki Kagei (Fukuoka) has investigated the global attractor for two-dimensional Bénard convection, using governing equations that include the heating effect of viscous dissipation. Maria Schonbek (Santa Cruz) has investigated the decay properties of magneto-hydrodynamic flow, showing that if the energy of the magnetic field is non-oscillating, then the energy of the velocity field decays to zero.

Three of the articles presented here take up fundamental issues concerning the linear Stokes equations. Hermann Sohr (Paderborn) and Maria Specovius-Neugebauer (Paderborn) give a new theory of the exterior Stokes problem using homogeneous Sobolev spaces. This is based in part on a new decomposition theorem for these spaces. Paul Deuring (Magdeburg) gives a new approach to the regularity of solutions of the Stokes equations in the neighborhood of conical boundary points. In particular, he supplies L_p -estimates. Werner Varnhorn (Dresden) constructs a solution of the Stokes resolvent equations by methods of hydrodynamic potential theory. This is based on a new explicit representation of the fundamental tensor.

Finally, in the direction of numerical applications, Reimund Rautmann (Paderborn) presents an article on a new convergence result for Rothe's method of approximating the Navier-Stokes equations. – In this place we have to thank Kerstin Wielage for fitting together all the contributions written in heterogeneous codes.

John Heywood, Kyuya Masuda, Reimund Rautmann, Vsevolod A. Solonnikov

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