Quantum Mechanics

Selected Topics

This page is intentionally left blank

Quantum Mechanics

Selected Topics

Askold M. Perelomov

Inst. of Theor. & Exp. Phys., Russia

Yakov B. Zel'dovich

P. L. Kapitsa Inst. of Phys. Problems, Russia

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

QUANTUM MECHANICS — SELECTED TOPICS

Copyright © 1998 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 981-02-3550-X

This book is printed on acid-free paper.

Printed in Singapore by Uto-Print

Contents

Preface	ix
Chapter 1. Discrete Spectrum	1
1. Introduction	1
2. States with small binding energy	8
3. Point interaction and its correspondence to boundary conditions ..	14
4. Particle in field of several point potentials	17
5. The Coulomb potential	20
6. Three-dimensional oscillator	28
7. Virial theorem and its generalizations	41
8. Identical particles and statistical physics	44
Chapter 2. Continuous Spectrum	49
1. Introduction. Wave functions of continuous spectrum with $l = 0$..	49
2. Motion with orbital angular momentum $l \neq 0$. Motion in the Coulomb field	57
3. Wave functions of continuous spectrum. Scattering cross-section ..	64
4. Optical theorem and its generalization	70

Chapter 3. Analytic Properties of Wave Function	75
1. Analytic properties of S-matrix	75
2. "Redundant" poles	84
3. Properties of residues of $S_I(k)$	88
4. Dispersion relations	93
Chapter 4. Inverse Scattering Problem	101
1. The Marchenko equation	101
2. Reflectionless potentials	106
3. Isospectral deformations of quantum oscillator	109
4. Isospectral deformations of the Schrödinger equation and invariants of the Korteweg-de Vries equation	114
Chapter 5. The Green Functions and Perturbation Theory ...	123
1. Introduction. The Green function of the radial Schrödinger equation	123
2. Regular method of obtaining of the Green functions	128
3. Some properties of the Green functions	134
4. The Green function for several free particles	136
5. Perturbation theory. Coordinate representation	138
6. Momentum representation	143
7. The Green function in momentum representation. Operator algebra.	149
8. Scattering operator	154
9. Formulae for point potentials	159
10. Perturbation theory for continuous spectrum	163
11. Convergence of series of perturbation theory	168
12. The time-dependent Green function	174

Chapter 6. Quasi-classical Approximation	177
1. Wave function in quasi-classical approximation	177
2. Quasi-classical approximation for the degenerate Fermi gas	182
3. Multi-dimensional case	188
4. Non-stationary problems	199
Chapter 7. Exact Solutions of Non-stationary Problems for Oscillator	213
1. Introduction	213
2. Wave function of oscillator with variable frequency under action of external force	214
3. Quantum oscillator under action of external force. Transition probabilities	218
4. Parametric excitation of quantum oscillator	220
5. Oscillator with variable frequency under action of external force. Transition probabilities	227
6. Quantum oscillator and adiabatic invariants	231
7. Quasi-energy of system under action of periodic force	236
8. The Heisenberg representation and canonical transformations ...	241
Chapter 8. Quasi-stationary States	255
1. Introduction. The Gamov theory	255
2. Wave functions	262
3. Example of quasi-stationary state	267
4. Decay of quasi-stationary state	272
5. Radioactive decay law	278
6. Generalization of normalization. Perturbation theory for quasi-stationary states	282
7. Asymptotic behaviour of wave function at $r \rightarrow \infty$ and $t \rightarrow \infty$...	285

viii *Contents*

8. Creation of unstable particle	289
9. Transition from quasi-stationary to stationary states	295
10. Collision time	298
11. Types of long-lived states	301
Appendix A. Specific cases of the Schrödinger equation spectrum	305
Appendix B. Quasi-classical properties of highly excited levels in the Coulomb field	317
Bibliography	321
Subject Index	333

Preface

Just as a "demographic explosion" gives much trouble to sociologists and economists, the problem of an "information explosion" is in the face of scientists and teachers in all its magnitude.

The difference between both problems is in the fact that the birth-rate of the population can be limited much more easily than the birth-rate of articles.

Only the editors of scientific journals dream about the Golden Age when authors themselves would criticize their own works and reject all of them except brilliant ones.

The birth of an article which represents even a small step forward from the generally accepted level of knowledge gives a satisfaction and a pleasure which the author cannot overcome. In our opinion, one should not try to fight against the "information explosion", but rather try to direct its energy, i.e. efforts of the huge army of scientists, into a common channel.

What is important for chemistry and zoology is perhaps the classification of information and the mechanization of a search for materials concerning a given chemical compound or biological species.

As far as theoretical physics, we believe that overviews and monographs summarizing the results of some studies in the fields of today are of utmost importance. For an overview, one should select without a bias the most valuable results from a large number of works.

As a matter of fact, text-books in which the material is revised as science is developed, pursue the same goal. In the preface to the famous Course of Theoretical Physics, L.D. Landau and E.M. Lifshitz state that learning of this Course provides the base which will be sufficient for understanding of original publications in journals.

One should note that in recent years there has been a certain gap between text-books and new original works. The present book aims to reduce this gap.

It is intended for to be intermediate between a course in quantum mechanics and a present-day study of a number of problems in atomic, nuclear physics and partly in physics of elementary particles.

This monograph is concerned with the following general physical problems.

1. Systems with a low binding energy, namely the deuteron and the negatively charged hydrogen ion.
2. Systems with the Coulomb potential — the hydrogen atom.
3. Unstable systems, such as radioactive nuclei, and autoionization states.
4. The detailed theory of the harmonic oscillator and its application to oscillations of the electromagnetic field in laser systems.

5. The ionization of bound states (a negative ion or an atom) by the field of a strong light wave (such as a laser light pulse focused by a lens).

The book also deals with some methods of theoretical physics which are discussed here in more detail than in most text-books. These are:

1. Analytic properties of the wave function and scattering matrix.
2. The Green function of the Schrödinger equation.
3. The quasiclassical approximation.
4. The inverse scattering problem.
5. Exact solutions of non-stationary problems.

It is not customary to point out in the preface of a book which problems are not touched upon, for the latter are difficult to limit, and besides this might prejudice to authors. Deviating from the tradition, we mention two problems which might be naturally expected to be included in a modern course of quantum mechanics, namely the Regge poles and the Feynman path integrals. Both questions are elucidated in the literature, and we consider it is reasonable to omit them.

The present monograph is a revised and extended version of a part of the book *Scattering, Reactions and Decays in Non-relativistic Quantum Mechanics* (Nauka: Moscow, 1971, in Russian) by A.I. Baz', A.M. Perelomov and Ya.B. Zel'dovich; namely of the part that was written by the second and the third authors. We recommend Chapters 8 to 11 of this book as an additional material.

Moscow, October 1987

A.M. Perelomov

Ya.B. Zel'dovich

Addendum

To my great regret, I had to complete this book without the participation of my teacher and co-author, the remarkable scientist and human being, Academician Ya.B. Zel'dovich.

The author of these lines has been learned much from Ya.B. Zel'dovich and feels a great obligation towards him. His untimely decease interrupted our joint work on the book. However, the general scheme for compiling of it made by Ya.B. Zel'dovich and pages written by him are remained unchanged, although on technical reasons there is a considerable delay in the publication.

Zaragoza, April 1998

A.M. Perelomov