Exact Analysis of Structures with Periodicity using U-Transformation

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U-TRANSFORMATION**

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PREFACE

This book introduces a new analytical method, U-transformation method, for the exact analysis of structures with periodic property.

In a broad sense, it is fair to say that the U-transformation method is originated from the mode method and yet it is different from the latter. In fact it may be called a mode subspace method. Performing U-transformation on a cyclic periodic structure is equivalent to projecting the displacement vector on the orthogonal mode subspaces in the unitary space. Hence the governing equations for a cyclic periodic structure and the continuity conditions between the substructures could be uncoupled. Consequently the governing equation in the U-space will be turned into N independent equations in N subspaces, where N is the total number of substructures. Besides, the order of the subspace is the same as the degrees of freedom of a single substructure. The difference in the N problems in the N subspaces may be reflected by taking a different value in one of the parameters in each subspace. In analytical method, that means the N governing equations need to be solved only once.

The U-transformation method has many advantages over the mode transformation method. When applying the mode method, it is necessary to work out the mode shapes of the structure. That is a task involving a lot of calculations and computing costs. In the end only numerical solutions would be obtained. In applying the U-transformation method, one can leave out calculating for the mode shapes in a periodic structure. This is because it is possible to give in advance some normalized orthogonal basis for each mode subspace. They form the columns of the U-matrix in the U-transformation. This is the physical background of the uncoupling of the governing equations and the continuity conditions of a periodic structure. For cyclic periodic structures, the U-transformation method has many obvious advantages over the numerical method. When the number of substructures in a periodic structure is altered, it will require a complete re-calculation to obtain a new solution by any numerical method. But with the U-transformation method, it is not necessary to re-analyze the problem again because the number of substructures is merely one of the parameters in the analytical solution. When there is a change in the number of substructures in a periodic structure, the expression of the analytical solution will not change.

By applying the U-transformation method for the analysis of cyclic periodic structures, either analytical approach or numerical approach may be adopted. However, the aim of this book is to illustrate how to obtain analytical solutions.

This book also provides many practical as well as theoretical exact explicit solutions for a large range of problems. Many of these solutions are new results that have just appeared in international journals for the first time together with some which have not been published before. The reason why the analytical method is used is not only because it will save computation costs, but also because it will enable the relationships between various parameters of the geometrical and physical properties in the problem to be reflected clearly. These relationships will probably often become the bases for optimum design or investigation. It is not a gross overstatement to say that analytical methods still have their great value to justify their existence even in this super-computer age of to-day. The analytical solutions given in this book will serve the following four distinctive purposes if not anything else:

1. It has been demonstrated that the U-transformation method can be applied to obtain exact analytical solutions for periodic structures. These exact analytical solutions could not be easily obtained or have not been obtained by other methods before. This book has illustrated how the U-transformation method can be used to derive the exact solutions for a number of problems of finite and infinite periodic structures under arbitrary static or dynamic loading.

2. With an exact analytical solution for a problem as a reference standard, it is possible to investigate the errors in approximate models for the problem. The analytical solution also gives the relations between the various parameters that contribute towards the errors. Hence it will enable a decision to be made as to under what conditions the approximate model may be acceptable. For instance, a continuous beam on periodic elastic supports may sometimes be simplified as a beam on elastic foundation. As another example, a stiffened plate with equal distance ribs may be treated as an orthotropic plate. This book has given detail analysis and discussions on the errors incurred by employing these two models. An explicit solution has also been given.

3. When a regular uniform continuous structure is sub-divided into a regular equal finite element mesh, for instance, a rectangular plate is sub-divided into a rectangular mesh, the sub-divided mesh may be treated as a periodic structure. That will allow it to be analyzed by the U-transformation method to yield an analytical solution for the finite element equations. Hence, that would facilitate the study of the rate of convergence as well as the convergence of the finite element solution when the number of finite elements approaches infinity. Furthermore, an explicit expression for the main body of the errors may be derived. This could not be achieved with the other methods.

4. It has been demonstrated how to apply the U-transformation method to the analysis of nearly periodic structures. Here only problems containing a few substructures with different properties are discussed and only for the case when

there is disturbance in the parameters of stiffness and masses. The aim is to study the mode localization phenomena. This phenomenon is very sensitive to the variation of a particular parameter in the sub-structure. It is therefore necessary to obtain a rigorous solution. If an approximate method is employed, it may lead to significant disagreement. It is best to employ analytical methods for this kind of problem.

The first five chapters in this book deal with the fundamental concept and theory of the U-transformation method. In addition to attempting to explain how the U-transformation method can be employed for the analysis of periodic structures, the authors of this book also try to explain how this method may be used in the study of two very important aspects of theoretical mechanics as mentioned in 3 and 4 above. Chapter 6 of this book deals with the exact solution of finite element equation. The chapter also gives an example for the case when the element stiffness matrix contains a stiffness regulative factor. It is possible to adjust the value of the stiffness regulative factor such that the error is minimum so that the rate of convergence may be enhanced. The mode localization is discussed in Chapter 7. Some solutions and results which have not been solved by using other methods are given in this chapter. These include the conditions that localized mode may occur in one dimension nearly periodic structures, and the theoretical results of the threshold value that causes the localized mode of 3-D nearly periodic structures.

As a mathematical tool, the U-transformation method has now been well systematized. The authors deeply believe that the problems that can be analyzed with the U-transformation method could be far more than those mentioned in this book.

The authors would like to express their sincere gratitude to the University of Hong Kong for providing research grants to support this work and to Professor J.K. Liu and Dr. Y. Li for their assistance in the preparation of this book and typing of the manuscript. While it is not possible to make individual acknowledgment to all the sources of references and materials adopted in this book, it is hoped that full credit is noted through this acknowledgment. Though every effort has been made to ensure exactness and to eliminate errors as far as possible, should errors be found, the authors would greatly appreciate comments and notifications from the learned readers.

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