

40 YEARS IN MATHEMATICAL PHYSICS

World Scientific Series in 20th Century Mathematics

Published

Vol. 1 The Neumann Compendium
edited by F. Bródy and T. Vámos

Vol. 2 40 Years in Mathematical Physics
by L. D. Faddeev

Forthcoming

Vol. 3 After Me Cometh a Builder
by Y. Manin

Vol. 4 A Mathematician and His Mathematical Work
edited by S. Y. Cheng, G. Tian and P. Li

40 YEARS IN MATHEMATICAL PHYSICS

L. D. Faddeev

*St. Petersburg Branch
Steklov Mathematical Institute
Russia*

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Faddeev, L. D.

40 years in mathematical physics / L.D. Faddeev.

p. cm. -- (World scientific series in 20th century mathematics ; vol. 2)

Includes bibliographical references

ISBN 9810221983. -- ISBN 9810221991 (pbk.)

1. Mathematical physics -- History. 2. Faddeev, L. D.

3. Physicists -- Russia (Federation) -- Biography. I. Title.

II. Series.

QC19.6.F33 1995

530.1'5--dc20

95-24959

CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

The author and publisher would like to thank the following publishers for their assistance and their permission to reproduce the articles found in this volume:

American Institute of Physics (*Sov. Phys. Usp.*), American Mathematical Society, Birkhäuser Publishers (*Sel. Math. Sov.*), Elsevier Science Publishers B. V., Harwood Academic Publishers, Johnson Reprint Corporation, Kluwer Academic Publishers Group, Plenum Publishing Corporation (*Theor. Math. Phys.*, *J. Sov. Math.*), Springer-Verlag

Copyright © 1995 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, Massachusetts 01923, USA.

Printed in Singapore.



L D Faddeev

L D Faddeev was born in 1934 in Leningrad, USSR (now St Petersburg, Russia), and his personal life and scientific career have been closely connected with that city. He graduated from Leningrad University in 1959 and has since been occupying a research position in the Leningrad (or St Petersburg) Branch of the Steklov Mathematical Institute of the Soviet (now Russian) Academy of Sciences. There he gradually established the

Laboratory of Mathematical Methods in Physics, which he has been heading until now.

The subject of Prof Faddeev's research reflects several new tendencies in modern mathematical physics, some of which were pioneered by him, such as quantization of the Yang–Mills field, scattering theory, and quantum theory of solitons. He is author or coauthor of 5 monographs and 150 papers.

Prof Faddeev was an invited speaker and/or lecturer at numerous international conferences, workshops and schools. He was President of the International Mathematical Union from 1986 to 1990, and is a member of the Russian Academy of Sciences and a foreign member of the National Academy of Sciences of the USA, the American Academy of Arts and Sciences, the Academies of Finland, Sweden and Poland, and Academia Europæa, as well as *Dr Honoris Causa* of the Universities of Paris, Uppsala, Buenos Aires and La Plata, and Nankai University. He has won the State Prize of Russia, the Danni Heineman Prize in Mathematical Physics of AIP, the UAP Prize in Science, and the Dirac Medal and Prize.

This page is intentionally left blank

CONTENTS

Introduction	ix
Comments on Paper 1	1
1. 30 Years in Mathematical Physics Proceedings of the Steklov Institute of Mathematics (American Mathematical Society, 1988), No. 3, pp. 3–28	3
Comments on Papers 2–4	29
2. Perturbation Theory for Gauge-Invariant Fields (with V. N. Popov) <i>Gauge Theory of Weak and Electromagnetic Interactions</i> , ed. C. H. Lai (World Scientific, 1981), pp. 213–233	31
3. The Feynman Integral for Singular Lagrangians <i>Theor. Math. Phys.</i> 1 , 1 (1970)	52
4. Covariant Quantization of the Gravitational Field (with V. N. Popov) <i>Sov. Phys. Usp.</i> 16 , 777 (1974)	65
Comments on Paper 5	77
5. Introduction to Functional Methods <i>Méthodes en théories des champs/Methods in Field Theory</i> — Les Houches, Session XXVIII, 1975, eds. R. Balian and J. Zinn-Justin (North-Holland, 1976), pp. 1–40	79
Comments on Paper 6	119
6. Inverse Problem of Quantum Scattering Theory. II <i>J. Sov. Math.</i> 5 , 334 (1976)	121
Comments on Papers 7–10	185
7. Quantum Completely Integrable Models in Field Theory <i>Mathematical Physics Reviews, Sec. C: Math., Phys. Rev. I</i> , ed. S. P. Novikov (Harwood, 1980), Vol. 1, pp. 107–155	187
8. The Quantum Method of the Inverse Problem and the Heisenberg XYZ Model (with L. A. Takhtadzhan) <i>Russian Math. Surveys</i> 34 , 11 (1979)	236
9. Integrable Models in (1+1)-Dimensional Quantum Field Theory <i>Développements Récents en Théorie des Champs et Mécanique Statistique/ Recent Advances in Field Theory and Statistical Mechanics</i> — Les Houches, Session XXXIX, 1982, eds. J.-B. Zuber and R. Stora (Elsevier, 1984), pp. 561–608	294

10. From Integrable Models to Conformal Field Theory via Quantum Groups <i>Integrable Systems, Quantum Groups, and Quantum Field Theories</i> — Proceedings of the NATO Advanced Study Institute and XXIII GIFT International Seminar on Recent Problems in Mathematical Physics, eds. L. A. Ibort and M. A. Rodríguez (Kluwer, 1993), pp. 1–24	342
Comments on Paper 11	367
11. The Search for Multidimensional Solitons Proceedings of the IV International Symposium on Nonlocal Field Theories (Nonlocal, Nonlinear, and Nonrenormalizable Field Theories), 1976	369
Comments on Paper 12	383
12. Hamiltonian Approach to the Theory of Anomalies <i>Recent Developments in Mathematical Physics</i> — Proceedings of the XXVI Int. Universitätswochen für Kernphysik Schladming, eds. H. Mitter and L. Pittner (Springer-Verlag, 1987), pp. 137–159	385
Comments on Paper 13	409
13. The Energy Problem in Einstein's Theory of Gravitation (Dedicated to the Memory of V. A. Fock) <i>Sov. Phys. Usp.</i> 25 , 130 (1982)	411
Comments on Paper 14	425
14. Lagrangian Mechanics in Invariant Form (with A. M. Vershik) <i>Sel. Math. Sov.</i> 1 , 339 (1981)	427
Comments on Paper 15	439
15. Einstein and Several Contemporary Tendencies in the Theory of Elementary Particles <i>Relativity, Quanta, and Cosmology in the Development of the Scientific Thought of A. Einstein (1879–1979)</i> (Johnson Reprint Corporation, 1979), Vol. 1, pp. 247–266	441
Comments on Paper 16	461
16. A Mathematician's View of the Development of Physics <i>Frontiers in Physics, High Technology and Mathematics</i> — Proceedings of the ICTP 25th Anniversary Conference, eds. H. A. Cerdeira and S. O. Lundqvist (World Scientific, 1990), pp. 238–246	463

INTRODUCTION

In September of 1955, I entered the last year of undergraduate studies in the Physics Department of Leningrad University. I was to prepare a diploma work, supervised by Prof Olga Ladyzhenskaya, for the Chair of Mathematics and Mathematical Physics. Contrary to the practice of almost all physics departments throughout the world, where the mathematical courses are provided by external professors (mostly from parallel mathematics departments), the Physics Department of Leningrad University organized the mathematical education on its own. This had already been done by academician V. I. Smirnov in the '30s, when he established the special Chair of Mathematics with the support of physicists, notably V. A. Fock. However, this Chair was considered an auxiliary one and did not have diploma practice till 1955. Then it got the name of Chair of Mathematics and Mathematical Physics and was allowed to have its own students, give special courses and supervise the diploma work. Thus I was really in the very first group of students, who in the following year, 1956, defended their diploma on the specialization in mathematical physics.

Forty years have passed. Looking back in time I can say that I followed a very satisfactory direction of scientific work. Mathematical physics has acquired a completely new form during the last 30 years. The evolution of quantum theory in application to elementary particles and high energy physics as well as to solid state physics, where we deal with the system of many (or infinite) degrees of freedom, required the use of highly sophisticated mathematical methods, many of which entailed new development. Precisely this goal is met by modern mathematical physics, which includes in its arsenal practically all branches of mathematics, besides the traditional domain of differential equations. And, without false modesty, I can say that I have made several decisive contributions to this development. In my scientific autobiography, written 10 years ago and included in this volume, I describe in detail the motivations, modes and results of my work.

The person delivering the prestigious Sir Run Run Shaw Lecture at the University of Stony Brook is given a memorable present. It is a coffee cup with an inscribed list of scientific achievements of the lecturer. I was invited by Prof C. N. Yang in March 1993 to present this lecture. My cup has the following list:

1. Three-body quantum-mechanical problem.
2. Quantization of the Yang–Mills field.
3. Three-dimensional inverse scattering problem.
4. Algebraic Bethe Ansatz.
5. Quantization with anomalies.
6. Leningrad–St. Petersburg group in modern mathematical physics.

Most of those results are reviewed in this volume. The exception is topic 1, on which there exist several monographs. Besides the technical papers, the volume

contains the scientific autobiography already mentioned above, an essay on the relation between mathematics and physics, and an article dedicated to the 100th anniversary of Einstein's birth. It is a matter of pride for me to give a list of my former students, many of whom belong to the group mentioned in point 6. Here they are: V. S. Buslaev, V. N. Popov, O. A. Yakubovsky, P. P. Kulish, I. Ya. Aref'eva, A. B. Venkov, V. E. Korepin, L. A. Takhtajan, A. E. Reiman, E. K. Sklyanin, N. Yu. Reshetikhin, V. O. Tarasov, F. A. Smirnov, S. L. Shatashvili, A. Yu. Volkov, A. Yu. Alekseev. I also collaborated intensively with B. S. Pavlov, A. A. Slavnov, S. P. Merkuriev, V. B. Matveev, A. R. Its, A. G. Izergin and I. V. Komarov, all of whom but Slavnov graduated from the Physics Department of Leningrad University. Quite a few of them are now working outside Russia, spreading the influence of our traditions.

I am very grateful to Prof K. K. Phua and World Scientific Publishing Company for the proposal to compile and publish this volume. Most of the papers were originally published in Russian and here the texts from the English translations of corresponding Russian journals are reproduced. The holders of copyrights for these translations have granted permission to use them free of charge and I am grateful to them. I did a little editing of the translations, with the goal of correcting some technical terms. Also, I used this opportunity to rectify some mistakes in Paper 6.