MATHEMATICAL LOGIC FOR COMPUTER SCIENCE

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MATHEMATICAL LOGIC FOR COMPUTER SCIENCE

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PREFACE

Many aspects in various branches of mathematical logic are related to computer science. This book describes those aspects which are related to each other, including first-order logic, constructive logic, and modal logic.

Materials adopted in this book are intended to attend to both the peculiarities of logical systems and the requirements of computer science, but those concerning the applications of mathematical logic to computer science are not involved. Each of the topics is discussed concisely with the essentials emphasized and the minor details excluded.

This book consists of an introduction, nine chapters, and an appendix. In the Introduction the nature of mathematical logic is explained. In Chapter 1 the basic notions of sets, inductive definitions and inductive proofs are reviewed. These are prerequisites for the remaining chapters. In Chapters 2 and 3, propositional and first-order logic are constructed, in which formal deducibility is treated in the form of natural deduction, resembling ordinary mathematical reasoning. Chapter 4 introduces another type of formal deducibility and discusses the relationship between it and the natural deduction system. Chapters 5 and 6 study soundness and completeness, some of their important applications (including compactness, Löwenheim-Skolem's Theorem, and Herbrand's Theorem), and some basic notions of model theory. Chapter 7 introduces constructive logic and discusses the relations between constructive and non-constructive logic. In Chapters 8 and 9, various systems of modal logic are investigated. In the Appendix, a simple form of formal proof in natural deduction is introduced.

This book is aimed at graduates, undergraduates, and researchers in computer science.

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