Nonlinear Integrals and Their Applications in Data Mining

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To our families

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Preface

The theory of nonadditive set functions and relevant nonlinear integrals, as a new mathematics branch, has been developed for more than thirty years. Starting from the beginning of the nineties of the last century, several monographs were published. The first author of this monograph and Professor George J. Klir (The State University of New York at Binghamton) have published two books, *Fuzzy Measure Theory* (Plenum Press, New York, 1992) and *Generalized Measure Theory* (Springer-verlag, New York, 2008) on this topic. These two books cover most of their theoretical research results with colleagues at the Chinese University of Hong Kong in the area of nonadditive set functions and relevant nonlinear integrals. Since the 1980s, nonadditive set functions and nonlinear integrals have been successfully applied in information fusion and data mining. However, only a few applications are involved in the above-mentioned books. As a supplement and in-depth material, the current monograph, *Nonlinear Integrals and Their Applications in Data Mining*, concentrates on the applications in data analysis. Since the number of attributes in any database is always finite, we focus on our fundamentally theoretical discussion of nonadditive set function and nonlinear integrals, which are presented in the first several chapters, on the finite universal set, and abandon all convergence and limit theorems.

As for the terminology adopted in the current monograph, words like *monotone measure* is used for a set function that is nonnegative, monotonic, and vanishing at the empty set. It has no fuzziness in the meaning of Zadeh's fuzzy sets. Unfortunately, its original name is *fuzzy measure* in literature. Word "fuzzy" here is not proper. For example,

words “fuzzy-valued fuzzy measure defined on fuzzy sets” causes confusion to some people. Such a revision is the same as made in book *Generalized Measure Theory*. However, in this monograph, we prefer to use *efficiency measure* to name a set function that is nonnegative and vanishing at the empty set, rather than using *general measure*. This is more convenient and intuitive, and leaves more space for further generalizing the domain or the range of the set functions. Hence, similar to the classical case in measure theory [Halmos 1950], the set functions that vanish at the empty set and may assume both nonnegative and negative real values are naturally named as *signed efficiency measures*. The signed efficiency measures were also called *non-monotonic fuzzy measures* by some scholars. Since, in general, the efficiency measures are non-monotonic too, to distinguish the set functions satisfying only the condition of vanishing at the empty set from the efficiency measures and to emphasize that they can assume both positive and negative values as well as zero, we prefer to use the current name, signed efficiency measures, for this type of set functions with the weakest restriction. Thus, in this monograph, we discuss and apply three layers of set functions named monotone measures, efficiency measures, and signed efficiency measures respectively.

The contents of this monograph have been used as the teaching materials of two graduate level courses at the University of Nebraska at Omaha since 2004. Also, some parts of this monograph have been provided to a number of master degree and Ph.D. degree graduate students in the University of Nebraska at Omaha, the University of Nebraska at Lincoln, the Chinese University of Hong Kong, and the Chinese Academy Sciences, for preparing their dissertations.

This monograph may benefit the relevant research workers. It is also possible to be used as a textbook of some graduate level courses for both mathematics and engineering major students. A number of exercises on the basic theory of nonadditive set functions and relevant nonlinear integrals are available in Chapters 2–5 of the monograph.

Several former graduate students of the first author provided some algorithms, examples, and figures. We appreciate their valuable contributions to this monograph. We also thank the Department of Computer Science and Engineering of the Chinese University of Hong

Kong, the Department of System Science and Industrial Engineering of the State University of New York at Binghamton and, especially, the Department of Mathematics, as well as the Art and Science College of the University of Nebraska at Omaha for their support and help.

Zhenyuan Wang
Rong Yang
Kwong-Sak Leung

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Contents

Preface	vii
List of Tables	xv
List of Figures	xvi
Chapter 1: Introduction	1
Chapter 2: Basic Knowledge on Classical Sets	4
2.1 Classical Sets and Set Inclusion	4
2.2 Set Operations	7
2.3 Set Sequences and Set Classes	10
2.4 Set Classes Closed Under Set Operations	13
2.5 Relations, Posets, and Lattices	17
2.6 The Supremum and Infimum of Real Number Sets	20
Exercises	22
Chapter 3: Fuzzy Sets	24
3.1 The Membership Functions of Fuzzy Sets	24
3.2 Inclusion and Operations of Fuzzy Sets	27
3.3 α -Cuts	33
3.4 Convex Fuzzy Sets	36
3.5 Decomposition Theorems	37
3.6 The Extension Principle	40
3.7 Interval Numbers	42
3.8 Fuzzy Numbers and Linguistic Attribute	45
3.9 Binary Operations for Fuzzy Numbers	51
3.10 Fuzzy Integers	58
Exercises	59
Chapter 4: Set Functions	62
4.1 Weights and Classical Measures	63
4.2 Extension of Measures	66
4.3 Monotone Measures	69
4.4 λ -Measures	74

4.5 Quasi-Measures	82
4.6 Möbius and Zeta Transformations	87
4.7 Belief Measures and Plausibility Measures	91
4.8 Necessity Measures and Possibility Measures	102
4.9 k -Interactive Measures	107
4.10 Efficiency Measures and Signed Efficiency Measures	108
Exercises	112
Chapter 5: Integrations	115
5.1 Measurable Functions	115
5.2 The Riemann Integral	123
5.3 The Lebesgue-Like Integral	128
5.4 The Choquet Integral	133
5.5 Upper and Lower Integrals	153
5.6 r -Integrals on Finite Spaces	162
Exercises	174
Chapter 6: Information Fusion	177
6.1 Information Sources and Observations	177
6.2 Integrals Used as Aggregation Tools	181
6.3 Uncertainty Associated with Set Functions	186
6.4 The Inverse Problem of Information Fusion	190
Chapter 7: Optimization and Soft Computing	193
7.1 Basic Concepts of Optimization	193
7.2 Genetic Algorithms	195
7.3 Pseudo Gradient Search	199
7.4 A Hybrid Search Method	202
Chapter 8: Identification of Set Functions	204
8.1 Identification of λ -Measures	204
8.2 Identification of Belief Measures	206
8.3 Identification of Monotone Measures	207
8.3.1 Main algorithm	210
8.3.2 Reordering algorithm	211
8.4 Identification of Signed Efficiency Measures by a Genetic Algorithm	213
8.5 Identification of Signed Efficiency Measures by the Pseudo Gradient Search	215
8.6 Identification of Signed Efficiency Measures Based on the Choquet Integral by an Algebraic Method	217
8.7 Identification of Monotone Measures Based on r -Integrals by a Genetic Algorithm	219
Chapter 9: Multiregression Based on Nonlinear Integrals	221
9.1 Linear Multiregression	221

9.2 Nonlinear Multiregression Based on the Choquet Integral.....	226
9.3 A Nonlinear Multiregression Model Accommodating Both Categorical and Numerical Predictive Attributes	232
9.4 Advanced Consideration on the Multiregression Involving Nonlinear Integrals	234
9.4.1 Nonlinear multiregressions based on the Choquet integral with quadratic core.....	234
9.4.2 Nonlinear multiregressions based on the Choquet integral involving unknown periodic variation	235
9.4.3 Nonlinear multiregressions based on upper and lower integrals	236
Chapter 10: Classifications Based on Nonlinear Integrals	238
10.1 Classification by an Integral Projection.....	238
10.2 Nonlinear Classification by Weighted Choquet Integrals	242
10.3 An Example of Nonlinear Classification in a Three-Dimensional Sample Space.....	250
10.4 The Uniqueness Problem of the Classification by the Choquet Integral with a Linear Core	263
10.5 Advanced Consideration on the Nonlinear Classification Involving the Choquet Integral	267
10.5.1 Classification by the Choquet integral with the widest gap between classes	267
10.5.2 Classification by cross-oriented projection pursuit	268
10.5.3 Classification by the Choquet integral with quadratic core.....	270
Chapter 11: Data Mining with Fuzzy Data	272
11.1 Defuzzified Choquet Integral with Fuzzy-Valued Integrand (DCIFI).....	273
11.1.1 The α -level set of a fuzzy-valued function.....	274
11.1.2 The Choquet extension of μ	275
11.1.3 Calculation of DCIFI	277
11.2 Classification Model Based on the DCIFI.....	282
11.2.1 Fuzzy data classification by the DCIFI	283
11.2.2 GA-based adaptive classifier-learning algorithm via DCIFI projection pursuit	286
11.2.3 Examples of the classification problems solved by the DCIFI projection classifier	290
11.3 Fuzzified Choquet Integral with Fuzzy-Valued Integrand (FCIFI)	300
11.3.1 Definition of the FCIFI	300
11.3.2 The FCIFI with respect to monotone measures.....	303
11.3.3 The FCIFI with respect to signed efficiency measures.....	306
11.3.4 GA-based optimization algorithm for the FCIFI with respect to signed efficiency measures	309

11.4 Regression Model Based on the CIII..... 319

11.4.1 CIII regression model..... 319

11.4.2 Double-GA optimization algorithm 321

11.4.3 Explanatory examples 324

Bibliography 329

Index 337

List of Tables

Table 6.1 Iris data (from ftp://ftp.ics.uci.edu/pub/machine-learning-databases).....	179
Table 6.2 Data of working times in Example 6.4.....	183
Table 6.3 The scores of TV sets in Example 6.5.....	184
Table 10.1 Data for linear classification in Example 10.1	241
Table 10.2 Artificial training data in Example 10.7	252
Table 10.3 The preset and retrieved values of monotone measure μ and weights b	259
Table 10.4 Data and their projections in Example 10.8	266
Table 11.1 Preset and retrieved values of the signed efficiency measure and boundaries in Example 11.4	293
Table 11.2 Preset and retrieved values of the signed efficiency measure and boundaries in Example 11.5	294
Table 11.3 The estimated values of the signed efficiency measure and the virtual boundary in two-emitter identification problem	297
Table 11.4 Testing results on two-emitter identification problem with/without noise ...	298
Table 11.5 The estimated values of the signed efficiency measure and the virtual boundary in three-emitter identification problem	299
Table 11.6 Testing results on three-emitter identification problem with/without noise	299
Table 11.7 Values of the signed efficiency measure μ in Example 11.13.....	318
Table 11.8 Results of 10 trials in Example 11.14.....	326
Table 11.9 Comparisons of the preset and the estimated unknown parameters of the best trial in Example 11.14.....	326
Table 11.10 Results of 10 trials in Example 11.15.....	327
Table 11.11 Comparisons of the preset and the estimated unknown parameters of the best trial in Example 11.15.....	327

List of Figures

Figure 1.1	The relation among chapters.....	3
Figure 2.1	Relations among classes of sets	15
Figure 3.1	The membership function of Y	25
Figure 3.2	The membership function of O	26
Figure 3.3	The membership function of $\bar{Y}$	30
Figure 3.4	The membership function of M	30
Figure 3.5	Membership functions of $\tilde{a}_b, \tilde{a}_w, \tilde{a}_f, \tilde{a}_g, \tilde{a}_e$	32
Figure 3.6	The α -cut and strong α -cut of fuzzy set Y when $\alpha = 0.5$	33
Figure 3.7	An α -cut of convex fuzzy set with membership function $m(x) = e^{-x^2}$	38
Figure 3.8	The membership function of $D+F$ obtained by the extension principle.	43
Figure 3.9	The membership function of a rectangular fuzzy number	47
Figure 3.10	The membership function of a triangular fuzzy number.....	49
Figure 3.11	The membership function of a trapezoidal fuzzy number.....	49
Figure 3.12	The membership function of a cosine fuzzy number.	50
Figure 3.13	Membership functions in Example 3.18	56
Figure 3.14	Membership functions in Example 3.19	57
Figure 5.1	The geometric meaning of a definite integral	125
Figure 5.2	The calculation of the Choquet integral defined on a finite set $\{x_1, x_2, x_3\}$	138
Figure 5.3	The chain used in the calculation of the Choquet integral in Example 5.7.....	139
Figure 5.4	The partition of f corresponding to the Choquet integral in Example 5.17.....	164
Figure 5.5	The partition of f corresponding to the Lebesgue integral in Example 5.18.....	166
Figure 5.6	The partition of f corresponding to the upper integral in Example 5.19.	169
Figure 5.7	The partition of f corresponding to the lower integral in Example 5.20.	170
Figure 5.8	The partitions corresponding to various types of nonlinear integrals in Example 5.21.....	173
Figure 7.1	Illustration of genetic operators	198
Figure 7.2	The flowchart of genetic algorithms	198

Figure 8.1 The lattice structure for the power set of a universal set with 4 attributes	211
Figure 10.1 The training data and one optimal classifying boundaries $x_1+2x_2 = 1.4$ with a new sample (0.3, 0.7) in Example 10.1	241
Figure 10.2 Interaction between length and width of envelopes in Example 10.2	243
Figure 10.3 The contours of the Choquet integral in Example 10.3	244
Figure 10.4 The projection by the Choquet integral in Example 10.3	245
Figure 10.5 A contour of the Choquet integral with respect to a signed efficiency measure in Example 10.4	246
Figure 10.6 Contours of the Choquet integral with respect to a subadditive efficiency measure in Example 10.5	247
Figure 10.7 Projection line and Contours of the weighted Choquet integral in Example 10.6	249
Figure 10.8 View classification in Example 10.7 from three different directions	260
Figure 10.9 The distribution of the projection $\hat{Y}$ on axis L based on the training data set in Example 10.7	261
Figure 10.10 The convergence of the genetic algorithm in Example 10.7 with different population sizes	262
Figure 10.11 Different projections share the same classifying boundary in Example 11.8	265
Figure 10.12 Two-class two-dimensional data set that can be well classified by cross-oriented projection pursuit	271
Figure 10.13 Two-class three-dimensional data set that can be well classified by cross-oriented projection pursuit	271
Figure 11.1 The α -level set of a fuzzy-valued function in Example 11.1	275
Figure 11.2 A typical 2-dimensional heterogeneous fuzzy data	284
Figure 11.3 The DCIFI projection for 2-dimensional heterogeneous fuzzy data	285
Figure 11.4 Illustration of virtual projection axis L when determining the boundary of a pair of successive classes C_{k_i} and $C_{k_{i+1}}$: (a) when $\hat{Y}^*(k_i) \leq \hat{Y}^*(k_{i+1})$; (b) when $\hat{Y}^*(k_i) > \hat{Y}^*(k_{i+1})$	288
Figure 11.5 Flowchart of the GACA	289
Figure 11.6 The training data and the trained classifying boundaries in Example 11.4	293
Figure 11.7 Artificial data and the classification boundaries in Example 11.5 — from two view directions	295
Figure 11.8 Relationship between $\tilde{f}$ and $\bar{f}_\alpha$	301
Figure 11.9 The membership functions and α -cut function of $\tilde{f}$ in Example 11.6	302
Figure 11.10 The membership functions of the Choquet integral with triangular fuzzy-valued integrand in Example 11.7	305
Figure 11.11 The membership functions of the Choquet integral with normal fuzzy-valued integrand in Example 11.8	306
Figure 11.12 Description of terminal ranges when μ is a signed efficiency measure	308

Figure 11.13 Correspondence in coding method.....	310
Figure 11.14 Distance definition on calculation of the left and the right terminals of $(C)\int \tilde{f}d\mu$	311
Figure 11.15 Membership functions of $\tilde{f}(x_1)$ and $\tilde{f}(x_2)$ in Example 11.11.....	315
Figure 11.16 The membership functions of $(C)\int \tilde{f}d\mu$ in Example 11.11	315
Figure 11.17 Membership functions of $\tilde{f}(x_1)$ and $\tilde{f}(x_2)$ in Example 11.12.....	316
Figure 11.18 The membership functions of $(C)\int \tilde{f}d\mu$ in Example 11.12	317
Figure 11.19 Membership function of $(C)\int \tilde{f}d\mu$ in Example 11.13.....	318
Figure 11.20 Structure of an individual chromosome in the double-GA optimization algorithm	322
Figure 11.21 Benchmark model in Examples 11.14 and 11.15.....	325