Homogenization

Serguei Kozlov (1954–1995)

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Homogenization

In Memory of Serguei Kozlov

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PREFACE

The theory of homogenization is very important in understanding the physical interactions on different length and time scales. An outstanding role in the creation of homogenization theory was played by Serguei Kozlov, a Russian mathematician whose untimely death at the age of forty interrupted a brilliant research career. This book is a tribute to him. It contains a biographical sketch of Serguei, the complete list of his publications, joint papers that were completed by his collaborators after his death, papers by several colleagues dedicated to him, and some recollections.

The editors thank Tania Semenova, Jane Berdichevsky, and Nadia Vasetsky for their help in preparing this book for publication.

V. Berdichevsky

V. Jikov

G. Papanicolaou

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SERGUEI KOZLOV — A BRIEF BIOGRAPHICAL SKETCH

Serguei Kozlov was born on May 17, 1954. He was the only child in the family. His father, the late Michael L'vovich Jakub, was a professor at the Moscow State University Law School; his mother, Ekaterina Ivanovna Kozlova, is still a professor at the Moscow Academy of Law.

In high school, Serguei decided to become a mathematician. He graduated with high honor from the Moscow State University, School of Mechanics and Mathematics (Mechmat), the best Russian educational and research center in mathematics and mechanics. In his student years, Serguei specialized in different equations. He chose Professor M. I. Vishik to be his advisor. He graduated from Mechmat in 1976, and stayed there as a graduate student. His Ph.D. thesis was concerned with the homogenization theory of differential operators with random and almost periodic coefficients. He completed his Ph.D. thesis in 1979.

After graduation, Kozlov was employed by the Department of Applied Mathematics of the Moscow Institute of Civil Engineering.

In 1989, S. Kozlov defended his Doctor of Science thesis "Homogenization of Random Structures". His scientific contributions are reviewed in this volume.

In 1991–1992, S. Kozlov had visited several universities in France. In 1992 he became a professor at the Aix-Marseilles University where he worked till his death. He committed suicide on April 7, 1995.

SOME RECOLLECTIONS

E.I. KOZLOVA:

I am glad to have an opportunity to describe Serguei's everyday life, although often it was only the background to his creative work.

We all claim our origins in childhood. Serguei's one was happy. He grew up surrounded by the attention and care of excessively loving parents. We strove to give a good up-bringing to our son — both in the moral and educational sense. We chose books for him to read, theaters and museums to attend, sports to participate in, etc., which were appropriate to his age.

Serezha ¹ very much loved being read to, and later to read himself. He especially liked the ballads about Robin Hood, which I sang to him and he could listen to them an infinite number of times.

He did not like fairy tales with an unhappy ending. Once, he was sitting in my lap and I was reading the "Little Red Riding Hood" to him. Already knowing how it ends, Serezha did not allow the wolf to do his mischief: knocking it of my hands, the book ended up in the opposite corner of the room.

Serguei was very stubborn as a child and also later in his life. If he wanted to take his favorite toy on a walk, a big metal dump truck, which rattled deafeningly when pulled along on the sidewalk, no pleading could dissuade him. I think this toy was known to all inhabitants of Sretenka (the street we lived on). If he did not want to enter a shop with me, I could not drag him in there. And any efforts to make him wear a warm coat if he thought it was not cold enough outside to wear it were wasted: he would throw the coat on the floor and run away.

However, along with being so obstinate, he was extremely law-abiding, which distinguished him from his peers. There was a theater next to our house, on the roof of which all the neighborhood kids walked and played, although it was forbidden. This prohibition was obeyed only by Serguei. In the autumn, when we gathered maple leaves, he only took the ones trampled by pedestrians from the sidewalk, because it was not permitted to walk on the grass; only rarely would he care-

¹The name "Serezha" is a less formal version of "Serguei" often used in everyday life.

fully pull an occasional leaf off the side of the lawn. This quality was probably inherited from his father, and I made constant efforts to overcome this inordinate attention to unimportant rules of conduct and to emphasize a sensible approach to them.

We strove to make him a person capable of saying "no", not unnecessarily submitting to the will of others, able of making independent decisions. To this end I often consulted him on serious problems and asked for his opinion even when he was a child.

One year before entering grade school, he began attending kindergarten to get used to socializing with kids his age.

Serezha learned to take care of himself rather early. Even when in grade school, he did not wake me up in the morning before leaving for school: he made his bed and fixed his breakfast himself. He knew how to cook an omelet and even managed to turn it over on the pan. When I was ill, it was Serezha and not my husband who took care of me: he went grocery shopping, cooked and served the meal. But Serezha was not a handyman. On the contrary, when something requiring handiness had to be done, he was quite awkward. In kindergarten, I could always distinguish Serguei's creations from those of other kids: a wobbly stool, an oddly embroidered napkin. As an adult, he was more adroit, but to hang up a painting, like Mark Twain, he would still require a few helpers.

Adolescence and youth changed him considerably. He quickly found a common language with his classmates and acquired many friends. He started spending more time outside with the kids, and he tried to smoke. Sometimes I was summoned to his school by the teachers to discuss "his misbehavior".

At home, he disliked talking about school, friends, or teachers. Sometimes I found out more about school in one hour from one of his friends than from Serguei during the whole year. Thus he began forming this characteristic which he later termed by the maxim: "Something unsaid can not be regretted". He stuck by this principle in the years to come.

We usually had good summer and other vacations. In June, when my husband and I worked, Serguei lived with my parents (and when little, also with a nanny) in the country in Kljaz'ma, near Moscow. There he had many friends, a bicycle, a river nearby. July we usually spent by the Baltic sea, August in Crimea by the Black Sea (more often

in Sevastopol' on the north side), or in Caucasus. On these vacations we would always lead an active style of life: we took long walks in the canyons and by ravine rivers and had picnics there. Serezha loved it all.

Serezha was a good but not a brilliant student. In middle school, he did not have any special preferences in academics. As for other activities, at some point he began collecting stamps, but without any system: he just liked buying them. For a short while he took up body building, but quickly lost the enthusiasm for it, although he continued to exercise with weights practically for the rest of his life. Physically, he was very strong.

Since our son did not have any particular leanings, my husband and I considered his following in our footsteps and becoming a lawyer, since he had potential. He possessed very clear logical thinking. In order to test his abilities, his father would sometimes present him with exceptional judicial cases from the textbook for university law department juniors. It was amazing that Serezha could impeccably solve these problems having at his disposal only the knowledge of the logical concept of the law and no practical experience. This choice of occupation also would have had the advantage of our being able to help him in preparation for the subject entrance exams. However, Serguei's fate ruled differently.

It came to happen that we changed residences and moved to a neighborhood of Moscow where the only good school in the area was that with an emphasis on mathematics. Serguei transferred to this school in the beginning of 8th grade.

He had to make significant mental efforts to catch up with his classmates who had already attended this school for a number of years. This endeavor was the beginning of his future choice of occupation. By the time of graduation he had decided for sure that he would attend the Moscow State University School of Mathematics and Mechanics (Mechmat).

In the new school he made new friends. Serguei was a good student, especially in science. During the last year of school, Serezha started preparing for the MSU entrance exams and was serious about his studies.

However, he was not able to enter Mechmat right away. He did well in his mathematics exams, but got a 'fail' in physics. It was a painful

blow both for Serguei and for us, more so for being so unexpected and undeserved. He was prepared exceptionally well in physics because during preparation he paid special attention to this subject. So many years have passed, but I still hear Serguei's voice over the phone: 'Mom, a fail.'

The Mechmat entrance exams were in July, those for other universities in August. The question of the second choice came up. My husband, afraid of further complications, insisted on the mathematics department of the Pedagogical Institute. I thought it would be a better to try the Moscow Institute of Electronic Industry since it had a good reputation. Finally, the latter was chosen. Serguei passed all their exams with distinction and was awarded a high stipend. The last exam was in composition. I remember the trembling with which Serezha and I approached the posted lists of those who failed, afraid to find the name "Kozlov" there. But the names on the list were so overwhelmingly of a particularly different kind that the applicants standing behind us could not abstain from connotative comments.

When he began attending the institute, Serguei studied with an enthusiasm which is easily explained by the upsets of the summer. During the first months he was selected to be part of the group with an intensive study of mathematics under the supervision of professor Maslov.

However, Serguei did not abandon his desire to study at Mechmat. His father petitioned the Rector of Moscow State University, academician Petrovsky, to transfer Serguei to Mechmat. According to the rules existing at that time, transfers were not permitted for freshmen, but the Rector promised to transfer Serguei at the beginning of the second semester if he passed all his first semester exams with distinction. Serguei complied with this condition and his wish came true: he became a student of Mechmat. The transfer procedure was not easy. It was necessary to obtain the permission for the transfer from his current Institute as well, which was difficult to accomplish. I was told that professor Maslov was very displeased by the fact that the administration allowed the transfer and thus robbed his group of their most talented student.

The difference of the first semester programs between the two universities was significant enough for Serguei to have to pass some additional exams in a few areas of mathematics, which he had to learn on his own, in a very short period of time. Serezha disliked talking about

his achievements, discussing his successes and repeating praises, even in the family, but he did admit that one of the instructors was very surprised by the fact that Serguei could master a specialized mathematical topic on his own. Usually, to the question of how something went, he had a ready answer: "OK."

Maybe this, second, exertion made him accustomed to mobilizing all his intellectual power, employing all his ability. During his study at Mechmat, he passed all his exams, without exception, with distinction. He was not an ordinary student: he attended theoretical seminars, eventually becoming their most active participant. He was rarely at home, spending most of the time in the libraries. After completion of his undergraduate studies, he entered graduate school and ahead of schedule received his Ph.D. under the supervision of professor M.I. Vishik. It so happened that on the day of Serguei's Ph.D. thesis defense, I received in the mail the permission to purchase a car, "Zhiguli" (in those times, it was impossible to buy a car — they were distributed according to one's place of employment). Thus, we had an opportunity to give Serguei a car as a gift for his successful defense. He quickly mastered it and became an aggressive, but very skillful driver.

His swift and successful dissertation did not safeguard Serguei from a new blow which was very painful to him, although he attempted to conceal that. He very much wanted and hoped to remain at Mechmat, but a position was not found for him. He had to independently seek employment, and eventually started teaching at the Moscow Institute of Civil Engineering.

He continued to be active in research and started attending conferences abroad. Serguei was an exceptionally hard worker. Whichever period of his life I recall to mind, I see him at his desk for hours, almost without interruption, day after day, year after year.

After the Doctor of Science defense, which took place in Leningrad, he began working in Paris for months at a time. I lived with his family there once for two happy months. Eventually, he moved to Marseilles where he worked for the rest of his life.

Serguei got married very early — at the age of 19, during his third year at the university — and since then lived separately from us. His wife, Natasha (born Il'gisonis) was an exceptionally good person. Serezha fell in love with her in 9th grade (they went to school together). Without exaggeration, she deserved to be described by the

following epithets: feminine, smart, hard-working, strong, energetic, efficient, a wonderful wife, mother, and daughter and, moreover, a talented chemistry researcher. She was loved by everybody. During their life together, she completely relieved Serguei of all the household chores. They had a very harmonious life. She died shortly before Serguei's death, and her untimely end shocked everybody and greatly affected Serguei.

They had two sons — Leonid (he is now 22, about to graduate from Mechmat) and Vladimir (14 years old, in 10th grade). They live with me now.

Judging by the letters, which were written mostly by Natasha (she took it upon herself to perform this task as well), they had a very interesting life. Natasha established scientific contacts in Marseilles and had a productive scientific life there. On week-ends they went to the country, to the beach, to theaters, or museums. Serezha's letters were very unique. He could expressively describe a town where he was, eloquently relate impressions from going to museums (which he never failed to do).

The summers they usually spent in Moscow (only once in Grenoble), living mostly in Kljaz'ma. Serezha loved the country. He showed himself to be competent at organizing reconstruction and expansion of the house. He liked to mow and sow. Almost every summer, along with Natasha and her brother, Serguei rode the motor boat on the Volga. He loved fishing, water skiing, gathering mushrooms, making camp fires. He was very sociable, liked parties, singing (although he had neither a musical ear nor a voice). He established contacts easily not only in his line of work, because he also knew how to find a common language with all kinds of people. He knew how to have fun, how to laugh, and had a sense of humor. Among his non-official photographs, it is difficult to find one where he is not smiling. This quality he had since his youth. Even in school the teachers complained of Serguei's making comments and jokes which made the whole class laugh. But even during vacations, he never abandoned his work. His colleagues often came to visit him to discuss scientific problems.

I cannot describe what I felt when I found out about Serezha's death. My older grandson and I flew to Marseilles to attend the funeral. There we were overwhelmed by the great number of things to be done. I was utterly helpless since I did not know French, and my grandson

did not speak it well, and we would have been lost if it weren't for the assistance of Serezha's colleague Mark Duran and his wife Annie. They helped us all along this difficult path having to do with necessary death papers, cremation, sorting Serguei's papers, documents, bills. They put us up, helped empty out the apartment, packed the things to be mailed to Moscow, did not leave us for a minute, and took us to the airport. Such consideration is never forgotten and I am infinitely grateful to this family.

Obviously, this does not say everything about Serezha. Every person is too complicated a world even to sketch it from all sides, not even taking into consideration the private internal areas, which belong to the person alone.

Despite all the pain caused by Serezha's death, I am grateful to him for having given me forty years of happy motherhood and two grandsons. He lived a short but a very rich life. There are many happy pages in his book, but there are also sad ones.

Serezha honored the graves of his ancestors. On his every visit to Moscow, he went to the cemetery where the deceased members of his family are buried. There he found his final rest. He was cremated in Marseilles and buried in Moscow at the Himkinskoe cemetery.

Requiescat in pace.

M.I. VISHIK:

Serguei Kozlov, an excellent mathematician with a strong character and interesting personality, is no longer with us.

Serguei came under my supervision during his third year at Moscow University and subsequently became one of my best students. He participated in my seminars, and I was Serguei's advisor for his undergraduate, as well as graduate, research. After brilliantly defending his Ph.D. thesis, Serguei went on working as a mature and creative mathematician.

Moscow State University was a crucial part of Serguei's mathematical biography. The first years of his active research career coincided with the golden years of our University. At that time, Moscow mathematicians were on the leading edge of many fields of mathematics, which enabled Serguei to communicate with a great number of outstanding scholars and he managed to absorb the best traditions of the University of that time. When Serguei was a third-year student, A.V. Fursikov, A.I. Komech and I set out to study in depth probability theory and random processes because at that time our research interests turned to statistic and stochastic solutions of the Navier-Stokes system and related partial differential equations. To this end, together with B.L. Rosovsky we organized a seminar on these subjects for the students of Moscow University. Serguei attended the seminar regularly and became an active participant. The main topic of his research during these years was nonlinear parabolic equations with random forcing (his best result in the field was a form of Girsanov's equation for stochastic equations which contained partial derivatives), and after some time he became a real expert in the probability and random processes theories and, what is more, the best among those who took part in the seminar. Serguei also attended my "big" seminar on partial differential equations and their applications. In due time he became one of the most brilliant participants and often presented his results in excellent talks. Serguei was an active participant all the time he lived in Moscow, but even when he moved to France, having gotten a professorship there, each time he came back to visit his relatives he gave a wonderful lecture at the seminar. I should mention that shortly before his tragic death, he gave a talk marvelous in both style and content. Serguei was a very well educated mathematician; he profoundly mastered many fields in math-

ematics. Besides the above mentioned seminars, Serguei also attended, although not as regularly, those of S. Molchanov and Ja. Sinai. During the seminars, he always listened carefully, asked the speaker sharp questions and made interesting comments. When Serguei was my graduate student, he was able to set up new problems and solve them by himself; all I had to do was discuss with him the many problems he had already solved.

Serguei got interested in the problem of homogenization after J.-L. Lions' lecture at Moscow Mathematical Society. He concentrated on the topic of homogenization of equations with stationary random coefficients. Later, the main subject of his research became the averaging problem for partial differential equations with coefficients with high frequency of oscillations and related problems from pure and applied mathematics. In his research, Serguei liked new, difficult and concrete mathematical problems, disliked completing other people's work, and hated to prove theorems of existence and come up with general theories. For example, he was very proficient in the theory of pseudo-differential operators, but did not want to work with it. He was ready to cooperate with other mathematicians, but valued very highly independence in scientific research (I remember that he once said that one of his friends would have become a brilliant mathematician if it were not for his professor's strong influence).

Serguei and his attractive wife Natasha were both 19 when they got married. Although, being a chemist, she had a career of her own, Natasha created wonderful conditions for Serguei's scientific work. She compensated Serguei's ardor by her warm personality, tact and culture. They were both very friendly and hospitable, and hosted many merry parties, which Serguei loved. They had two sons. The younger one I knew rather closely, having made his acquaintance in Paris. He is a very talented boy; he went to school in Paris, wrote poetry in French and was one of the best in his class.

Serguei was adventurous: he loved to drive at high speed and rode a motorcycle around Marseilles. In the summer, riding a motorboat and water-skiing were favorite activities; in the winter, he went cross-country skiing in areas remote from Moscow suburbs. During one of these ski trips, on which he went with Serguei Kuksin, one of his skies broke. It started snowing heavily and it was getting dark, and, not having a compass, they lost their way. In deep snow and without one

ski, Serguei soon was exhausted. Fortunately, they came across a small village where they were able to spend the night. All his relatives and friends, including me, were extremely worried when he did not come back from his skiing trip. But the next day he finally returned, cheerful as always.

Serguei was very fond of his parents' dacha in Chelyuskino, near Moscow. He began its renovation just before leaving for France and was planning, upon his return, to work there in relative isolation, coming to Moscow only to teach. Unfortunately, these dreams became a reality only for a short time.

I shall keep him in my memory as an enjoyable companion, talented mathematician and a devoted friend.

SERGUEI KOZLOV — A REVIEW OF SCIENTIFIC CONTRIBUTIONS

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V. JIKOV¹

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Serguei Kozlov was a scientist of outstanding and versatile talent. All his achievements were distinguished by considerable professionalism and bright individuality. Research interests of Kozlov were related to the homogenization theory, disordered structures, Ito equations, as well as other problems of mathematical physics and probability theory. He had many remarkable results in averaging of difference schemes, random walks on lattices and Fokker-Planck equation. However, the major contributions are in the homogenization of quasi-periodic and random operators. At the moment when he entered the field, the asymptotic theory of averaging of periodic operators was developed. Many of the participants of this breakthrough tried unsuccessfully to understand how to do averaging for random structures and what might be an analogy of the "cell" problem for such structures. It was Serguei Kozlov who found the answer [4, 6].² This was a remarkable breakthrough, shedding a new light on many old results accumulated for more than a hundred years in various areas of mechanical engineering and physics, and putting many of them into the final form.

Being a lecturer he was an initiator and a leader of various scientific seminars. With his participation every seminar became more vital and interesting. With his exceptional talent to find the most perspective mathematical aspects in complicated engineering problems, Kozlov paid great attention to modern branches of science. He initiated scientific interests and activities of his colleagues to the problems in these areas, many colleagues are much obliged to him for helpful and deep discussions and support.

¹In English literature another spelling of the second author's name, Zhikov, is also used.

²References are made here to the list of publications of Serguei Kozlov which follows this review.

Serguei shared the point of view that there are two types of scientists: donors and recipients. In this classification, Serguei was certainly a donor. He threw out remarkable ideas. Many of them had become the basis or the starting points of further investigations. A much greater number of papers could have his name as an author, but he never agreed to be a co-author if his own contribution did not seem to him greater than that of the other participants. That is why his ideas and results appeared sometimes in papers having his name in the paragraph of acknowledgments only. Nobody heard, however, a word of reproof from him in this respect.

As for himself, Serguei applied a very high criteria of honor. Usually, his part in joint papers included either the basic points or the majority of details. He always had to ensure that his contribution was sufficient to warrant his name as co-author.

During his student years, Serguei studied Ito's partial differential equations. He investigated conditions for coincidence of measures generated by Ito's equations. Also he investigated ergodic properties of solutions and proved corresponding generalization of Girsanov's formulae. Later he returned to the matter again and again.

Since 1976 he was thrilled with a new topic, homogenization, which became the major theme of his research. His above-mentioned breakthroughs [4, 6] were followed by a series of outstanding papers. We note here his paper [9] "Homogenization and G-convergence of elliptic operators" written with Jikov, Oleinik and Kha Tien Ngoan. This paper was a milestone in the homogenization theory. Its annual citation index is still one of the highest in the field. This paper was a starting point and a guide manual for many mathematical studies.

In 1979-1983 a series of papers were written by Kozlov, some of them jointly with Jikov and Oleinik, on the homogenization and G-convergence of parabolic operators. At the same time Kozlov studied some asymptotic properties of Green functions in the homogenization problems.

In 1980 Serguei with Jikov and Zlotnik considered a model integrand that corresponds to "Sobolev spaces with varying exponents". Later this approach was developed in connection with the Lavrentiev effect in variational problems.

A problem of great interest in the homogenization theory is averaging of equations which contain randomness in the terms with low order

derivatives. Kozlov developed an approach to this problem that was based on his results on reducibility of operators with periodic and almost periodic coefficients. Later, in 1988-1992, he investigated jointly with Piatnitsky the asymptotic behavior of effective turbulent diffusion in the case of small viscosity coefficient.

A significant contribution by Kozlov was his approach to the averaging of difference equations and random walk on lattices. He developed some general homogenization methods and established the main results like the compactness theorem for difference schemes. With respect to random walks he obtained various sets of sufficient conditions for the central limit theorem. In part these results were established jointly with S. Molchanov. This work relates to the period of 1984-1990. It initiated a lot of further investigations both in mathematical and applied setting.

Always Serguei was interested in spectral problems for operators of the homogenization theory and in particular, with various spectral asymptotics. In this area, his results on multidimensional spectral asymptotics should be mentioned. Also he had proved jointly with M. Shubin a coincidence of spectra for random elliptic operators.

Some investigations of Kozlov in the period of 1987-1994 were concerned with the problem of effective conductivity and related topics for disordered structures. In this problem the rigorous mathematical methods and engineering approaches are mutually useful. Kozlov's contribution to this problem, especially his results on percolation, on the homogenization of highly perturbed media and hierarchical structures, were highly appreciated by mathematicians and physicists. In these activities he cooperated with K. Golden, A. Bensoussan, A. Beliaev, G. Panasenko, A. Bourgeat, L. Berliand and L. Malozemov. Each paper carrying the name S. Kozlov immediately attracted great attention from specialists and always initiated numerous investigations which resulted in developments of his original ideas.

In 1991 Kozlov had a provisional position at the Moscow Institute of Water Problems. There he dealt with applied problems of porous flows. This field was new for him, but in short period Serguei had found the way to connect his mathematical interests and abilities with problems of hydrodynamics. This subject was one of the main topics of his further researches in University of Marseille in the last 3 years of his life.

In hydrodynamics Kozlov studied a mathematical problem of homogenization for fluid flows in random porous media. Earlier this problem had been considered for periodically perforated domains by L. Tartar, G. Allaire and others. An extension of these results to more realistic random structures was an important challenging problem of the homogenization theory. Kozlov gave a rigorous justification of Darcy's law under some probabilistic assumptions about the porous structure. A principal trouble in comparison with the periodic case is caused by the percolation properties of random medium. Another one relates to porous domains containing pores of arbitrarily large size. In this case, the permeability of the porous medium could be equal to infinity. Kozlov suggested sufficient conditions for convergence in terms of a version of Poincaré's inequality. Then he proved this inequality under some assumptions on probabilistic distribution of geometric parameters like pore sizes and so-called "chemical lengths". Later these ideas were generalized by Kozlov's colleagues for non-steady flows in random media and for some other problems of homogenization.

In connection with viscous flows in porous media Kozlov's attention was attracted by the spectral properties of Stokes operator in a randomly perforated domain. For the Laplace operator with Dirichlet boundary conditions in a random domain containing a large number of "holes" the asymptotics of spectrum bottom was studied by A.-S. Sznitman. He used, in the spirit of Donsker-Varadhan approach, a method that relates to probabilistic interpretation of the Laplacian, and proved an explicit formula for the leading term of the principal eigenvalue. That way cannot be applied to vector operators. Kozlov proposed to use a re-scaling procedure which was justified by means of direct variational methods. In this way, the results on asymptotics of the principal eigenvalue were re-established for the Laplace operator. Later, Kozlov's method was extended for more general operators. His ideas are also useful for some non-linear problems corresponding to asymptotics for random structures.

For many years the homogenization of elliptic operators in perforated domains in connection with applied problems of diffusion and percolation was in the center of his interests. In 1985 he had suggested jointly with Jikov a conjecture that the connectedness of a perforated domain is the unique sufficient condition for convergence in the homogenization problem. Later this conjecture was justified for a certain class

of operators, and this result was extended and generalized to homogenization on fractals. The great interest in the latter kind of structures had appeared quite recently, and Kozlov was one of those who foresaw the importance of this area. His publication on this matter is concerned with the Serpinsky gasket. He obtained some explicit formulae and general homogenization results for that structure.

In things appertaining to other contribution of Kozlov, it is necessary to note his joint results with M. Yor on asymptotics of random number of Wiener walk, the so-called Wiener football. Also he dealt with KAM theory in relation with homogenization of nonlinear equations with almost periodic coefficients and homogenization of Fokker-Planck equation, which is more realistic than the elliptic equation for some applications.

Kozlov always strove to obtain explicit results and explicit formulae every time it was possible. In particular, he made a certain progress in studying the low-concentration asymptotics and sharpening the Maxwell formula for effective conductivity. Also he considered the asymptotic behavior of the Laplace-Dirichlet integrals and had described this problem in terms of differential geometry. A lot of explicit formulae for homogenized coefficients were justified by this approach.

Kozlov was a many-sided person, and everybody enjoyed interacting with him. He was extremely open-hearted and generous in sharing his new ideas. Collaboration with him was fruitful and pleasant for people of different ages. With his superlative talent, inexhaustible energy and wonderful capacity for work, Serguei participated in a great number of investigations. For many younger mathematicians he was a teacher and an advisor. The memory of Serguei Kozlov will be kept in the hearts of those who knew him.

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