Matrix Theory

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DAVID W. LEWIS

*Department of Mathematics
University College Dublin
Ireland*

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PREFACE

This book provides an introduction to matrix theory, and aims to provide a clear and concise exposition of the basic ideas, results and techniques in the subject. It combines the algebraic and analytic aspects of matrix theory. It presumes no knowledge beyond school mathematics although some familiarity with elementary calculus would be helpful in a few of the applications. It is hoped that the book can be profitably used by a wide range of students, including students of mathematics, engineering, science, and other disciplines where matrices arise. Complete proofs are given, although some are relegated to appendices at the end of chapters. This should enable the book to be used both by students who want all of the theory and those who are mainly interested in learning the techniques. The text is interspersed with many examples, applications and numerous exercises for the reader. Students who have already done an introductory linear algebra course may use the later chapters for a more advanced course.

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