Complex Vector Functional Equations

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To the authors' sons Branko Risteski and Svetoslav Covachev

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Preface

Complex vector functional equations are a new field in the theory of functional equations. Their development up to now has not been particularly dynamical. As every new mathematical discipline, complex vector functional equations call for their more detailed investigation. From this standpoint, in the last few years we dedicated a series of papers to these equations. Our multi-annual experience and the results obtained by the research in this new area show us the necessity to write a strictly scientific systematized monograph.

The object of this monograph is to give an overview to the recently obtained results of the authors from all aspects of utilization, which is obviously not possible with partial results in papers published in distinct mathematical journals.

We have divided our exposition on the field of complex vector functional equations in two parts which are devoted respectively to linear and nonlinear equations.

Part One consists of five chapters.

In the first chapter the general classes of cyclic functional equations in a complex vector form are considered. Among these are: the basic cyclic functional equation, the derived cyclic functional equation, the semicyclic functional equation, the special cyclic functional equation and the condensed cyclic functional equation.

In the second chapter the complex vector functional equations with operations between arguments are considered. First an operator functional equation is solved. Also, in this chapter the generalized equation of operator

type is solved. Next, some simple functional equations and functional equations with distinct functions are solved. Of this type of functional equations we can mention Fréchet's functional equations, to which an adequate place in this chapter is given. Finally, some complex vector functional equations with two operations between arguments are also solved.

The functional equations with constant parameters are considered in the third chapter. The equations of this type which are solved here are: the general parametric functional equation, the special parametric functional equation, the expanded parametric functional equation and the general expanded parametric functional equation.

The linear complex vector functional equations with constant coefficients are considered in the fourth chapter by a new matrix approach. In particular, we pay attention both to homogeneous and nonhomogeneous functional equations with constant coefficients. Finally, a paracyclic functional equation with complex constant coefficients is solved.

In the last fifth chapter of Part One two types of systems of functional equations are studied. First some systems in which each equation contains all unknown functions are solved, and then some systems in which not all equations contain all unknown functions.

Part Two is also divided in five chapters which are described below quite briefly.

Chapters 6, 7 and 8 are devoted respectively to quadratic, modified quadratic and expanded quadratic equations. Higher order complex vector functional equations are considered in the ninth chapter. Finally, in Chapter 10 systems of nonlinear (quadratic and higher order) complex vector functional equations are studied.

The present monograph is intended for mathematicians, physicists and engineers who use functional equations in their investigations.

The authors are well aware that all aspects of the complex vector functional equations cannot be included in an adequate way in a newly written book, and they will be grateful for all remarks and suggestions which will be valuable for the correction and completion of a subsequent edition.

The second author acknowledges partial support by Grant MM-706 with the Bulgarian Science Fund. A part of the book was written during the work of the second author at Fatih University, Istanbul, Turkey.

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