QCD SPECTRAL SUM RULES

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QCD

SPECTRAL SUM RULES

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PREFACE

The aim of this book is to provide :

- 1) an elementary introduction to the method of QCD Spectral Sum Rules (QSSR) for non-specialists and experimental physicists ;
- 2) a specialized review of QSSR developments over the last ten years of activity in this field.

The book is intended as an extension of and complement to my previous report in Ref. 34). In the first two chapters, I give a short general discussion of chiral symmetry, perturbative QCD and different QCD-like non-perturbative approaches other than QSSR. Chapters 3 and 4 are devoted to the theoretical foundations of QSSR and to different methods of evaluating the Wilson coefficients of the QCD condensates. The remaining chapters are concerned with different phenomenological applications of QSSR in QCD. Finally, in Chapter 18, I discuss the extension of QSSR to the composite models of electroweak interactions.

However, owing to the space-time limitation, I have been obliged to be selective in my discussion of QSSR methods and the hadronic channels. The former discussion is focused on the Laplace Sum Rule (LSR), the Finite Energy Sum Rule (FESR) and the Moments Sum Rules which are complementary methods. For the latter, I have selected the hadronic channels relevant for an understanding of chiral symmetry and the dynamics of light- and heavy-quark systems, for a non-perturbative test of QCD such as gluonia and for subsequent experimental research, e.g. the search for exotic states.

Most of our discussion of the phenomenology of QSSR is based on the optimal estimates extracted within the sum rule variable stability criterion and the value of the onset of the QCD continuum fixed either from FESR and/or from a stability criterion. Thus, in the course of this book it will be noticed that real progress has been made in improving the previous QSSR results by freeing them from handwaving and ad hoc phenomenological arguments. As a consequence, a coherent picture of hadron dynamics emerges from QSSR and it is impressive to see how QSSR fix the properties of the lowest ground state *plus* the correlated value of the onset of the QCD continuum by the few perturbative and lowest dimension non-perturbative parameters of the naive and the somewhat disappointing QCD Lagrangian.

Some other interesting problems naturally remain to be tackled before the picture is complete. However, in the present state of the art where our control of non-perturbative effects is very poor and while we await a new generation of Lattice Gauge Theory results, QSSR appear to be a powerful and competitive semi-phenomenological approach to the complex dynamics of hadrons.

Little further initial comment is required. R.P. Feynman's remark about gauge theories (Omni magazine 1979) sums up my feeling about QSSR :

"... A few years ago, I was very skeptical..

I was expecting mist and now it looks like ridges and valleys after all..."

to which I would add a Malagasy saying :

"Vary iray no nafafy, ka vary zato no miakatra"

which means : with one grain of rice sowed, one can gather by the thousand.

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