LATTICE GAUGE THEORIES

An Introduction

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LATTICE GAUGE THEORIES

An Introduction

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*To my wife Renate
and to my children Stefan, Christine and Laura
who taught me so many things about life*

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PREFACE

This book is based on a one year course I held at the University of Heidelberg and on a series of lectures I gave at the "Autumn College on Techniques in Many-Body Problems" at Lahore, Pakistan, in November of 1987. These lectures have been published in the proceedings to this school by World Scientific (Rothe, 1989). I was later encouraged by the editors of World Scientific to expand on the material presented at the autumn College. This I have done in this book.

The purpose of my lectures at Lahore was to introduce lattice gauge theories to young physicists who may not have the opportunity to attend a course on this subject at their home universities. I had therefore kept the discussion as elementary as possible, including only enough technical details to enable the reader to follow the published literature on this subject. In this book I have expanded substantially on the material presented at Lahore, and have included a number of technical details which I felt would be very helpful to those readers who may want to carry out research in this branch of elementary particle physics. I did, however, arrange the material in such a way that those physicists who are mainly interested in getting a bird eyes view on the subject can safely skip the technical parts, without the danger of getting lost at a later stage. This concerns, in particular, the discussion in sections 4 and 5 of chapter 4 on lattice fermions, and the weak coupling expansion in lattice quantum chromodynamics (QCD), chapter 14. I have included this material for the readers convenience, since it is not discussed in such detail in the literature. I also decided to include a chapter on the path integral formalism, since the entire book is based on the path integral approach to quantization, and I do not assume that everybody is familiar with this formalism. Those readers that have never come in touch with the path integral formulation of quantum field theory may find this chapter a bit technical. However, the results we derive, of which we will make ample use in this book, are very simple, and are easily understood by everybody.

This book is mainly addressed to graduate students interested in particle physics. But it is also of interest to physicists actively engaged in research in the field of lattice gauge theories, and who may want to get a more general view on this subject. It assumes that the reader has a fair background in quantum field theory. A moderate knowledge of the continuum formulation of quantum chromodynamics would certainly be very helpful. Also physicists

working in statistical mechanics may profit from reading this book, since the lattice formulation of field theories resembles closely that of complex statistical mechanical systems.

The book is divided into two parts. In the first part, comprising chapters 1 to 16, I discuss the zero temperature formulation of field theories on a space-time lattice, and in particular QCD. They are the lattice analogues of the usual continuum field theories discussed in standard text books. The second part, consisting of chapters 17 to 20, deals with finite temperature field theory. The emphasis will be on QCD, but I shall use a scalar field theory to introduce to the reader a number of new concepts which play an important role in finite temperature QCD.

Since the main goal of this book is to stimulate the readers interest in this fascinating branch of elementary particle physics, I have taken an optimistic stand-point, selecting some results of Monte Carlo calculations which illustrate the phenomena in a particularly dramatic way. I did not attempt to present a critical analysis of the results, and have left it to the reader to confer the original literature. Nor did I attempt to give a complete list of references, which the reader can find in the numerous proceedings to lattice conferences. More detailed discussions of most of the topics presented in this book can be found in the proceedings to various schools. An introduction to lattice gauge theories can also be found in the monograph by M. Creutz: *Quarks, Gluons and Lattices*, published by Cambridge University Press (1983).

Hopefully this book will stimulate some of the readers to carry out some research in the field of lattice gauge theories. If so, I have achieved the purpose it has been written for.

I like to take this opportunity to thank a number of colleagues for their constructive criticisms and for having read several chapters of this book. In particular I am grateful to A. Actor, I. Bender, D. Gromes, F. Karsch, K. H. Mütter, I. O. Stamatescu and W. Wetzel. I am especially grateful to W. Wetzel for having checked a number of formulae, and for his extensive technical help in getting the manuscript into its final form. I also want to express my gratitude to Mrs. U. Einecke and Mrs. M. Steiert for having typed so patiently the manuscript in $\text{\TeX}$. Finally, I am particularly thankful to my family, whose continued support has made this book possible. In particular my children had to dispense of their father for many (!) hours.

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