SOME ELEMENTARY GAUGE THEORY CONCEPTS

Born in Guangzhou, China, and brought up mostly in Hong Kong, Dr Chan Hong-Mo obtained his PhD in Birmingham, England. Before joining the Rutherford Appleton Laboratory and settling in Oxford, he was on the staff of CERN, Geneva, for nearly ten years, besides having been a member of the Institute for Advanced Study, Princeton, and a visiting professor at SUNY Stony Brook and the University of Washington, Seattle. His research interest in particle physics includes both theoretical and phenomenological topics.

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The two authors' joint work on this volume about elementary gauge theory concepts has been a labour of love to which they have devoted a number of years.

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Preface

Gauge theory, which underlies modern particle physics as well as the theory of gravity, and hence all of physics as we know it today, is itself based on a few fundamental concepts, the consequences of which are often as beautiful as they are deep. Unfortunately, in view of the pressure to cover aspects of the theory that are necessary for its many important applications, very little space is usually devoted in textbooks and graduate courses to the treatment of these concepts. The present little volume is an attempt to help in some small degree to redress this imbalance in the literature.

Our aim is to make these concepts and some of their immediate consequences accessible to all physicists, including graduate students, and no effort has been spared to make the whole volume understandable to the nonspecialist, given sufficient effort. The reader is warned, however, that the topics covered are elementary only in the sense of being fundamental, not in the sense of being shallow or easy. Although all will already feature at the classical field level, and most even before the introduction of an action principle, they often lead one to pose questions of some profundity which are still at the forefront of research or still unresolved. We believe, however, and we hope the reader will agree, that the subject matter is of sufficient interest to merit the effort in unravelling it. The selection of topics included may be a little personal, but for this we proffer no apology, for it is only those subjects on which we ourselves have worked do we feel confident in presenting with sufficient clarity. Obviously, this little volume is not meant to be a comprehensive work by any means. To us, it is a labour of love, a pleasure to share with others what we think we have understood, and a challenge to explain in simple terms what are at times rather difficult abstract ideas.

Our approach is physical but we shall have no hesitation in introducing mathematics when it is a help to the understanding. In the presentation, little previous knowledge is assumed of the reader apart from what is normally taught in graduate schools, but neither has there been any conscious attempt to avoid or trivialize essential difficulties and intrinsically abstruse concepts.

Since the material contains the wisdom of many, accumulated over decades,

it is difficult, if not altogether impossible, to ascribe to individuals any but just some few specific items. Rather than attempting to do so, therefore, we shall quote references only chapter by chapter and only in general terms, giving not a comprehensive list, of which we are incapable, but only a list of those few particular works from which we ourselves have learned the most. We apologize to those contributors whom we should have but have not quoted; the omission is due not to wilful neglect but to ignorance on our part.

We are deeply indebted to Professor C.N. Yang for first interesting us and then in teaching us much of what we know in the subject. We have also greatly benefited from Professor G. Segal for advice, especially on the mathematics of loop space, and from Dr. P. Scharbach for a most enjoyable collaboration on monopole dynamics. Further thanks are due to Dr. R. Newman and Ms J. Faridani for a critical reading of parts the manuscript, to Mr. S. Williams for help in $\text{\LaTeX}$, and lastly to our publisher World Scientific for their indulgence in extending our deadline time after time.

Chan Hong-Mo
Tsou Sheung Tsun

Oxford, February 1993

N.B. We use the metric $(+ - - -)$ and put $\hbar = c = 1$.

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