INTRODUCTION TO NONLINEAR DYNAMICS FOR PHYSICISTS

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INTRODUCTION TO NONLINEAR DYNAMICS FOR PHYSICISTS

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Preface

This is a series of lectures on nonlinear dynamics for physicists. The level is that of an advanced undergraduate or beginning graduate student. The assumption is made that the student has completed an advanced mechanics course and has had some exposure to mathematical physics of the sort encountered in an advanced electricity and magnetism or quantum mechanics course. No detailed knowledge of nonlinear dynamics is assumed, but familiarity with ordinary differential equations is. Some of the material is cast at a level which is more challenging to persons having only an undergraduate background in Physics but we felt that the subject was sufficiently interesting and rich that posing this kind of challenge was within the scope of the course. Similarly some of the material is aimed below that of a typical first year graduate student, but we also felt that this would serve both as a reminder and as a presentation of perhaps a different point of view for such students.

In any case the main goal of the lectures is to present both substantial qualitative information about phenomena in nonlinear systems to the advanced Physics student making the flavor tantalizing, yet providing sufficient quantitative material that the student would learn how to proceed in similar situations without the instructors "magic wand", which seems, we admit, to be waved here and there. Thus we set ourselves an impossible challenge overall, but we hope that the reader will find some success in our achieving these goals. Anyway, we trust enough success that we can hope for feedback from readers on topics (both those included and those left out), difficulty of presentation, and other items related to scientific material and pedagogy.

We hope that the reader, and even more the person who both hears the lectures and read these notes as they evolve to more finished stages, will have addressed three main questions about nonlinear dynamics:

- What is nonlinear dynamics all about and what makes it differ from linear dynamics which permeates all familiar textbooks?

- From the physicist's point of view, why should we study nonlinear systems and leave the comfortable territory of linearity?
- How can one progress in the study of nonlinear systems both in the analysis of these systems when we know them and in learning about new systems from observations of their experimental behavior?

Of course, neither these lectures nor whole lifetimes of work can possibly answer all of these questions in the finest detail. Nonetheless, we have set these impossible goals so that the student will reach with us slightly beyond an easy grasp.

These lectures were originally given by M. I. Rabinovich as Physics 155 in the UCSD Physics Department during the Fall Quarter of the 1992/93 academic year. Notes were taken and then transcribed by M. M. Sushchik, and edited by H. D. I. Abarbanel. It is our plan that this course will be taught by Abarbanel in the 1993/94 academic year at UCSD, and that they will be improved based on that experience and extensive input from the students of each course. Eventually we are thinking of making these notes into a monograph/textbook, but the barriers for turning a useful pedagogical product into a polished book are known to be high.

The problems were not part of the course when it was first taught, but they have been added by us as a guide for the reader, and we do plan to incorporate them in subsequent presentations of the material. We actively solicit feedback from readers of these notes. They are meant to be unpolished, yet, hopefully, transmit substantial material which is of interest to Physicists.

Henry D. I. Abarbanel
M. I. Rabinovich
M. M. Sushchik
January, 1993

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