FLUCTUATIONS AND LOCALIZATION

IN MESOSCOPIC ELECTRON SYSTEMS

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to Hanne, Mathias and Stephan

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Preface

This book is about quantum mechanics in macroscopic systems. The central themes are quantum fluctuations and localization by quantum interference. Usually, quantum phenomena like interference and tunneling show up in the microscopic world of atoms and molecules, not on macroscopic scales of cars and houses. However, there is no fundamental difference between microscopic and macroscopic scales in quantum mechanics. The reason for the absence of these effects in our macroscopic world is that individual objects lose their quantum mechanical *phase coherence* after some time while interacting with many thermalized objects. But as soon as the temperature decreases this time increases and in cold materials of the real-world, for example in metals and semiconductors, quantum phenomena become dominant. Such *mesoscopic* systems contain a huge number of particles and the holistic nature of quantum mechanics becomes evident already in electronic measurements. Due to randomness in the systems the measured quantity fluctuates as a function of applied fields in an unpredictable, yet reproducible way. Despite this *fingerprint character* of fluctuations their statistical properties are universal, *i.e.* they are the same for a large class of different mesoscopic systems, having only very few parameters in common.

Localization of electrons is a dramatic effect of destructive interference. As a consequence a metal can become an insulator while reaching mesoscopic scales. The localization-delocalization transition is a critical phenomenon driven by quantum fluctuations. This is in contrast to usual phase transitions that are driven by thermal fluctuations. Nevertheless, the theoretical description may be similar. Accordingly, in the beginning

of mesoscopics a one-parameter scaling theory was formulated stressing the role of *conductance* as the (one-parameter) scaling variable. The localized and delocalized phases are separated by a critical point determined by a critical value of conductance. In this sense, the scaling variable conductance plays a similar role as temperature in ordinary critical phenomena. In an equilibrium critical phenomenon like spontaneous magnetism the order parameter is a macroscopic observable like magnetization. It is finite in the "ordered phase" and vanishes in the "disordered phase". Therefore, one likes to know such order parameter for the localization-delocalization transition, too. However, the notion of an order parameter was not fully clarified in the early localization theory. Furthermore, the one-parameter scaling theory has been questioned once it was noticed that physical quantities fluctuate strongly and average values are not characteristic for the resulting broad distributions. Over the years it turned out that one-parameter scaling theory does work in terms of *typical values* of the local density of states and of the conductance which form the order parameter and scaling variable of the localization-delocalization transition. Below a certain length scale, called the correlation length, local quantities are strongly fluctuating. Furthermore, fluctuating local quantities look self-similar when they are plotted as a "random landscape" over real space. This so called *multifractal* behavior becomes universal on approaching the localization-delocalization transition where the correlation length becomes macroscopic. This multifractal behavior resolves the puzzle about the order parameter. Although the *average* local density of states is finite in both localized and delocalized phases, its *typical* value distinguishes between both phases in a unique way: In the localized phase the local density of states is typically zero and in the delocalized phase it is typically finite.

Based on elementary quantum and statistical physics this text points to the universal characteristics of fluctuations and on the localization mechanism. General concepts and methods are stressed such as scaling laws for distribution functions. Tools from condensed matter theory are used flexibly. Involved technical details are skipped to reach a broad overview on the field including topics like quantum dots, the quantum Hall effect and some recent developments. The text has three levels of sophistication. The first two chapters give a broad overview about phenomena and offer qualitative explanations with little formalism. They can be read independently of the rest as a brief review on the topic. The third chapter offers a more systematic approach to modeling and transport theory in mesoscop-

ics. It may prove to be helpful for studying other topics not addressed in this book. Here it is meant to prepare for the discussion of fluctuations and localization following in the remaining three chapters. In these a course is taken from idealized systems over more realistic systems to the localization-delocalization transition.

The potential reader is a graduate student in theoretical physics, who likes to begin working in the field of mesoscopic physics, experimentalists working in a special area of mesoscopics, and theorists who are not experts in the field, but like to learn about it without diving into many technicalities. I hope that even some experts might find interesting perspectives.

The book grew out of a review article [Janssen (1998)] and several short lecture courses on related topics that I held at different places since 1998. I tried to give a brief, but self-contained presentation. The established tools of diagrammatics and field theory are touched in the text, but mainly I concentrated on those subjects which are not so popular in other books on mesoscopics, for example network models and real-space renormalization.

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