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R. P. Agarwal

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PREFACE

World Scientific Series in Applicable Analysis (WSSIAA) aims at reporting new developments of a high mathematical standard and of current interest. Each volume in the series shall be devoted to mathematical analysis that has been applied, or potentially applicable to the solutions of scientific, engineering, and social problems. This volume of WSSIAA contains 47 research articles on Inequalities by leading mathematicians from all over the world and a tribute by R.M.Redheffer to Wolfgang Walter on his 66th birthday to whom this volume has been dedicated.

R.P.Agarwal

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