From Order to Chaos

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FROM ORDER TO CHAOS

Essays: Critical, Chaotic and Otherwise

Leo P. Kadanoff

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FROM ORDER TO CHAOS

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