ATTRACTORS OF QUASIPERIODICALLY FORCED SYSTEMS

WORLD SCIENTIFIC SERIES ON NONLINEAR SCIENCE

Editor: Leon O. Chua
University of California, Berkeley

Series A. MONOGRAPHS AND TREATISES

- Volume 1: From Order to Chaos
L. P. Kadanoff
- Volume 2: Ideal Turbulence
A. N. Sharkovsky, E. Yu. Romanenko, and M. B. Vereikina
- Volume 3: Dynamics of Piecewise-Monotone Maps
A. N. Sharkovsky, S. F. Kolyada, and A. G. Sivak
- Volume 4: Qualitative Tools of Nonlinear Dynamics
Part 1: Local and Global Bifurcations
L. O. Chua, A. L. Shil'nikov, L. P. Shil'nikov, and D. V. Turaev
- Volume 5: Qualitative Tools of Nonlinear Dynamics
Part 2: Homoclinic Chaos
L. O. Chua, A. L. Shil'nikov, L. P. Shil'nikov, and D. V. Turaev
- Volume 6: Stability, Structures and Chaos in Nonlinear Synchronization Networks
V. S. Afraimovich, V. I. Nekorkin, G. V. Osipov, and V. D. Shaf'eev
Edited by *A. V. Gaponov-Grekhov and M. I. Rabinovich*
- Volume 7: Smooth Invariant Manifolds and Normal Forms
I. U. Bronstein and A. Ya. Kopanskii
- Volume 8: Dynamical Chaos: Models, Experiments, and Applications
V. S. Anishchenko
- Volume 9: Frequency Domain Methods for Nonlinear Analysis: Theory and Applications
G. A. Leonov, D. V. Ponomarenko, and V. B. Smimova
- Volume 10: Critical Phenomena and Roads to Chaos in Multiparameter and Multidimensional Nonlinear Systems
A. P. Kuznetsov, S. P. Kuznetsov, and I. R. Sataev
- Volume 11: Nonlinear Dynamics of Interacting Populations
A. D. Bazykin
- Volume 12: Attractors of Quasiperiodically Forced Systems
T. Kapitaniak and J. Wojewoda

Series B. SPECIAL THEME ISSUES AND PROCEEDINGS

- Volume 1: Chua's Circuit: A Paradigm for Chaos
Edited by *R. N. Madan*
- Volume 2: Complexity and Chaos
Edited by *N. B. Abraham, A. M. Albano, A. Passamante, P. E. Rapp, and R. Gilmore*
- Volume 3: New Trends in Pattern Formation in Active Nonlinear Media
V. Perez-Villar, V. Perez-Munuzuri, C. Perez Garcia, and V. I. Krinsky

ATTRACTORS OF QUASIPERIODICALLY FORCED SYSTEMS

Tomasz Kapitaniak and Jerzy Wojewoda

Technical University of Łódź
Łódź, Poland

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

ATTRACTORS OF QUASIPERIODICALLY FORCED SYSTEMS

Copyright © 1993 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 27 Congress Street, Salem, MA 01970, USA.

ISBN 981-02-1525-8

Printed in Singapore.

Contents

Preface	vii
1 Introduction	1
2 Attractors of Dynamical Systems	3
2.1 Basic Attractors	3
2.2 Poincaré Map	9
2.3 Lyapunov Exponents and their Calculation	12
3 Strange Nonchaotic Attractors	15
3.1 Examples	15
3.1.1 Quasiperiodically Forced Van der Pol Oscillator	15
3.1.2 Quasiperiodically Forced Chemical System	22
3.1.3 Quasiperiodic circle map	25
3.2 Prediction of Non-Chaotic Strange Attractors	28
3.2.1 Existence of Quasiperiodic Solutions	28
3.2.2 Hopf Bifurcation in the Averaged System	35
3.3 Distinction Between Strange Chaotic and Strange Nonchaotic Attractors	38
3.4 Strange nonchaotic transients	43
3.5 Models of Strange Nonchaotic Attractors	49
3.5.1 Geometrical Model	49
3.5.2 Dynamical model	55
4 Inhibition of Chaotic Behaviour in Coupled Geophysical Models	58
4.1 Chaos and Aperiodicity in Geophysical Models	58
4.2 The Salzman Oscillator	61
4.3 El Nino - Southern Oscillation	65
5 Experimental System with Dry Friction	71
5.1 Experiments	71
5.2 Properties of the Power Spectra	73
5.3 Behaviour of the Friction Force	78
5.4 Model for the Observed Behaviour	80
References	84
Index	91

This page is intentionally left blank

Preface

In this book we discuss the influence of quasiperiodic force on dynamical systems. With this type of forcing different types of attractors are possible, for example strange nonchaotic attractors which have some unusual properties.

The main part of this book is based on the authors' recent works, but it also presents the results which are the combined achievements of many investigators.

We would like to acknowledge the discussion remarks of M. S. El Nashie, J. Brindley and R. Barron which helped us in the preparation of this book.

Finally, we wish to thank our wives Margaret and Alice for their patience.