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FROM CHAOS TO ORDER

Methodologies, Perspectives and Applications

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To Our Parents

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Foreword

The realization at the start of this decade that chaotic systems can often be controlled by smart butterflies excited many people, and the famous Ott-Grebogi-Yorke paper pointing out the possibility was the seed for a large body of research which is still growing rapidly. The point is that if we ask the right sort of questions, questions which may differ from those normally asked by a control theorist, we may be able to get a chaotic system to do something desirable with rather little control effort on our part. I call this control by smart butterflies, because it depends on the infamous butterfly effect, which says that chaotic systems are sensitive to small changes. Suppose that rather than saying “the system must do so-and-so”, we try to find some acceptable behavior among the infinitely many interesting things the system is already capable of doing: if we succeed in finding such a behavior mode, we can turn sensitivity to our advantage by making very small adjustments to steer the system into the mode we have chosen.

This differs from the usual control viewpoint, where we tend to choose some simple behavior such as stable equilibrium, or tracking a reference signal to some desired accuracy. The existing tools of control theory are well-suited for getting that sort of result, and to the extent that the control community took an interest in chaotic systems, it was originally to point out the unastounding fact that enough negative feedback could make chaos go away: controls people tended to think of suppressing chaos rather than using it. There are very good reasons for this, of course, but the fact that the control and dynamical systems communities had fundamentally different aims led to some misunderstandings, in which control theorists saw the dynamical systems work as naive and dynamical systems workers thought that control theorists only knew how to control using elephants instead of butterflies. Both of these extreme views were wrong, and in any case there were exceptions, with some dynamicists trying to learn control theory, and several practicing control engineers being quick to recognise the possibilities. Mathematical biologists were also quick off the mark. Perhaps practicing engineers and biologists are so used to dealing with mucky problems that they find it easy to contemplate changing one's control objectives to suit the system, instead of changing the system to suit one's control objectives.

Among control theorists, the most prominent exceptions were perhaps Chen and Dong, who were almost unique on either side of the debate in taking

the trouble to listen carefully to both sides, and learn the techniques from the other camp well enough to be able to integrate them into their own work. Anyone who has even begun to read in this area will be aware of their research papers, and also of their survey papers, which have been excellent places to start exploring this area.

As far as I know, this book is the first attempt outside conference proceedings to attempt a broad-ranging presentation of both the essential background and the many new results in control of chaotic systems and related areas. It covers an amazing amount of ground, from introductory material on nonlinear dynamics and on standard control methods, through the important areas of OGY-type control, targeting, synchronization, entrainment and so on, to aspects of system identification and modeling, and “intelligent” controllers. The advanced area of controlling spatio-temporal chaos has a chapter to itself. At the end of the book there are even some first ideas on how to make non-chaotic systems chaotic if that proves desirable.

Some dynamical systems workers have made sweeping claims about the desirability and even the inevitability of chaos in social, economic, evolutionary or environmental systems. Even if one does not accept such claims (which are not made by Chen and Dong), the ease of production of chaos in electronic systems; its prevalence in the very foundations of Newtonian mechanics, with the solar system known to be chaotic; the difficulties it causes in electro-optical systems such as lasers, and its everyday appearance in mechanical systems—we have all met vibrating machinery, and perhaps even ridden in old vehicles that vibrate irregularly — all these examples mean that controlling chaotic systems is a serious endeavor and that powerful results could have enormous consequences throughout science and engineering.

Understanding how to control chaotic systems is therefore both exciting and rewarding. It is also very difficult, with a family of problems we more or less understand, a herd of problems that we only have the slightest idea how to tackle, and surely an entire ecosystem of problems that we haven't even discovered yet. For now, and undoubtedly for a considerable time to come, there is a definitive place to start understanding these problems, and that is this book.

Alistair Mees

November 1997

Perth, Australia

with a butterfly on my window and a storm just past.

Preface

When a local power failure occurred a year ago, the subsequent loss of six separate transmission lines eventually caused a power outage in seven states serviced by a massive power grid in the western region of the United States. Possibly the largest of its kind in the world, the utility grid is a complex web of interconnected power lines that stretches from Canada to Mexico, and from the Pacific Coast to the western edge of the Rockies. Fortunately, the weekend power collapse was no more than a momentary disruption in service for most people, but it is a good example that underscores the unpredictable yet inevitable consequence of the intrinsic nonlinearity and growing complexity of systems such as power networks — as system dynamics evolve, numerous influences affect each other in a nonlinear process, and there comes a point where small problems accumulate in ways that designers could not predict and would never intend.

Prominent among efforts to understand practical engineering problems like this, bifurcation and chaos theories seem to provide some clues. In fact, in the last two decades we have come to realize that rather than being the exception, chaos is the norm in many real systems. From the dynamics of physiological processes to chemical reactions, from panel flutter in supersonic aircraft to semiconductor laser, chaos has left its mysterious fingerprint almost everywhere. Yet, the consolation is that people have now started to look for ways to deal with these kinds of phenomena.

In the practice of science and engineering, aside from the desire to identify, describe, and understand chaotic phenomena in physical systems, a growing interest has been centering on the question of whether it is possible to control chaos and, if so, how. Until just a few years ago this thinking was only an esoteric specialty in academic communities. Now, it has become a fast-emerging research topic with practical implications. This monograph is about the quest for answers to the control of chaotic nonlinear systems and processes.

Like many technological advances in history, chaos control has not emerged as a burgeoning research field without suffering some misconceptions. These come from several sides. For a while since the concept of chaos first took shape, the prospects of controlling it were considered counterintuitive. Indeed, at one time the extreme sensitivity of a chaotic system to initial conditions seemed to suggest that its erratic motion is, in general, neither predictable nor control-

lable. This early doubt has now pretty much given way to the new reality of today; that controlling chaotic dynamics is no longer some wishful thinking or untried technology. On the other hand, although chaos control is a legitimate topic and should be treated as such, its *raison d'être* has occasionally faced skepticism, or rather, the cynicism that it is no different from classical control. We want to emphasize that established control theories mainly deal with linear systems which are never beset by chaos, and conventional nonlinear control methods only target certain classes of nonlinear systems but do not specifically address chaotic phenomena. It is now known that feedback control of chaos is not any harder than controlling other nonlinear systems — if suppression of chaos is the only concern. But without resorting to a brute force strategy or drastic modification of the original system, controlling chaos remains a difficult task today.

We subscribe to the idea that any viable theory should be one that is constantly evolving and developing. When merged, the twin virtues of classical control theory and advanced nonlinear science may bring about unexpected benefits. For chaotic systems, if we can incorporate control mechanisms that exploit certain defining characteristics of chaos, the approach becomes more interesting and, hopefully, more efficient and effective. In addition, due to the inherent association of nonlinearity and chaos to various issues, the scope of chaos control is much more diverse. When compared to other conventional approaches, chaos control has its unique features with regard to such aspects as objectives, perspectives, problem formulations, and performance measures.

We have sought to provide this monograph as a useful platform for illustrating some of the above points in the course of introducing various chaos control methods. Rather than trying to fulfill an unrealistic expectation that a universal theory of chaos control is in the making, we present in this book technical options — what approaches have been proven to be effective and what else is possible — and convey the philosophies involved. Seeking to instill a systematic approach to the subject of chaos control, we cover in a common framework a broad spectrum of methods based on various perspectives. We believe this to be a step forward in the direction of a serious treatment of a subject as important as chaos control.

Considering the ubiquity of chaos in the physical world and the vast yet rapidly growing array of information available on related topics, we have not intended this monograph to be exhaustive, even though it turns out to be quite extensive. We are essentially concerned with various perspectives, methodologies, and some potential applications. The approaches covered are representative, rather than inclusive, and we only include those techniques that allow us

to illustrate what we consider as their most salient features. We introduce the more established methods for chaos control; we also provide pointers to some related developments, referencing some works as recent as the past year.

The nature of the subject matter easily affords this monograph a multidisciplinary scope. We hope that as a side effect we facilitate a collective understanding of a wide range of available approaches. In fact, some of the techniques from different disciplines may be used in combination to achieve significant improvements in control performance. Knowing that the monograph would suffer from an ambitious goal of attempting to explain every technique with microscopic intensity, we try to strike a balance between depth and breadth, intentionally omitting trenchant conclusions in most of the case studies. We do, however, attempt to offer some pertinent and objective points of view on controlling chaos in an open and impartial manner. For example, we have taken the liberty to comment on the intentions behind certain efforts and to evaluate their merits and implications. We also attempt to speculate about generalizations or applications, if not to predict, outright, possible ongoing or future developments in this exciting and challenging field.

Henry Poincaré said, "*Il n'y a pas de problèmes résolus, il n'y a que des problèmes plus ou moins résolus.*" (There are no solved problems; there are only more-or-less solved problems.) Accomplishment notwithstanding, chaos control as a profound research subject is far from being complete. In fact, we think what Poincaré said remains a worthy reminder reflecting the current state of the art in chaos control. We consider the material presented in this monograph as part of a larger, ongoing effort in searching for effective means for controlling chaotic physical systems. We aspire that our monograph be not only retrospective, but more importantly, also both conducive and prospective.

Guanrong Chen
Xiaoning Dong
August, 1997

Houston, USA
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