Plastic Bending

Theory and Applications

SERIES ON ENGINEERING MECHANICS

Editor: W. Johnson, FRS, F. Eng.

Published

Vol. 1: When It Comes to the Crunch: The Mechanics of Car Collisions
N. W. Murray

Plastic Bending

Theory and Applications

T X Yu

Hong Kong University of Science & Technology

L C Zhang

University of Sydney

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co Pte Ltd

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Yu, T. X. (Tongxi X.), 1941–

Plastic bending : theory and applications / T.X. Yu, L.C. Zhang.

p. cm. -- (Series on engineering mechanics ; vol. 2)

Includes bibliographical references and index.

ISBN 981022267X

1. Plastic analysis (Engineering) 2. Plasticity. I. Zhang, L.

C. (Liangchi) II. Title. III. Series.

TA652.Y82 1996

620.1'1233--dc20

95-53979

CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

The authors and publisher would like to thank the following publishers for their permission to reproduce figures:

American Society of Mechanical Engineers

Cambridge University Press

Elsevier Science Ltd.

Mechanical Engineering Publications Ltd.

Oxford University Press

The Institute of Materials

The Society for Experimental Mechanics, Inc.

Copyright © 1996 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA.

Printed in Singapore.

To Our Families

This page is intentionally left blank

Foreword

It is a pleasure to be able to recommend this volume on *Plastic Bending* by two authors who have deeply researched the subject in Cambridge, England and Beijing, China. The present work is largely a collection of their researches in plastic bending, comprehending both structural and metal-forming plasticity. It contains an abundance of new results in nine chapters with two further chapters being devoted to numerical methods. As well as theoretical results the authors include much experimental verificatory work. Particularly interesting is the work on stamping circular and rectangular plates into dies, paying especial attention to springback phenomena so that plate of a required radius may be supplied to customers. As well, the complications and the generation of multi-point bending when pressing sheet or plate into dies, is a topic only recently reported on.

This volume is comprehensive and well-written with a great many references. There is no need to outline the scope of this text, as the book well gives a taste of it in the Introductory Chapter. Post-graduate students should find the volume of great value, as it is (I believe) the first work wholly given over to bending. The book will also commend itself to those engaged in the productive or manufacturing industry, since some problems are here attacked, in theory, for the first time.

It is a delight to read about plastic bending in so many new contexts because the subject has long needed a "face-lift".

W. Johnson

6 September 1995

This page is intentionally left blank

Preface

Plastic bending and stamping are widely used operations in sheet metal forming. Various machine parts, structural members and other items can be produced by these operations to provide shapes with single or double curvatures. In recent years, these operations have also been applied to form curved sheets made of non-metal materials, e.g. thermoplastics and thermoplastic matrix composites. Besides, plastic bending is encountered in many other engineering fields, such as structural crashworthiness and energy absorbing systems.

Despite its wide applications, plastic bending is one of the topics which has been almost neglected in theoretical studies for many years. As pointed out by Professor W. Johnson, FRS, bending is an area "where a deeper understanding of the elasto-plastic mechanics of the formed material in machines is needed", and "little of this work is yet available in book form for use in metal-forming plasticity." He concluded, "a new look at plastic bending is timely and this information for widespread use by international audiences is much needed." (*Journal of Materials Processing Technology*, **25** (1991), pp.289-290)

The present book is a monograph aimed to illustrate, from the mechanics point of view, the theory of plastic bending and its applications; most of the contents of the book derive from the research work carried out by the authors and their collaborators in the past 15 years. It is hoped that it will serve as a guide for research workers and post-graduate students who are interested in this subject, and as a reference for engineers and designers who need theoretical tools to solve their problems.

The text of the book contains an Introduction, 11 Chapters and an Appendix. The Introduction attempts to describe briefly the engineering background of plastic bending and the relevant mechanics problems. Chapter 1 outlines the Engineering Theory of Plastic Bending. This theory is based on a series of simplifying assumptions and has been widely adopted due to its simplicity. The more accurate theory, the Mathematical Theory

of Plastic Bending, is reviewed in Chapter 2. This theory is then used to discuss the range of applicability of the Engineering Theory of Bending. An appropriate mathematical description of large elastic-plastic deflection of beams (strips) is of great importance in theoretical modelling of bending processes. This is treated in Chapter 3, together with analytical solutions of typical problems.

Based on the fundamental theory presented in Chapters 1 - 3, Chapters 4 and 5 deal with plastic bending of beams (strips) where single curvature develops. Experimental studies and theoretical modelling of cylindrical bending of strips between punches and dies are discussed in Chapter 4; whilst Chapter 5 highlights the numerical methods for single-curvature bending.

Chapters 6 - 9 are devoted to the study of double-curvature bending and stamping of plates (or sheets). Chapter 6 is concerned with axisymmetric bending of circular plates in elastic-plastic range, where various loading conditions and solution procedures are examined. Chapter 7 concentrates on pressing circular plates into hemispherical dies, which is more complex than the axisymmetric bending because the contact between punch and plate varies with the punch travel. The study of double-curvature bending and stamping is extended to rectangular plates in Chapter 8, where experiments are compared with the predictions of theoretical models. Chapter 9 focuses on numerical methods for double-curvature bending and emphasis is put on the modified adaptive Dynamic Relaxation (maDR) method.

Wrinkling is a particularly important problem for plates undergoing double-curvature forming, because it often leads to failure of the forming products. This topic is investigated extensively in Chapter 10, where experimental, theoretical and numerical treatments are combined. From a mechanics point of view, wrinkling is a result of plastic buckling and post-buckling of plates and shells. To assist the reader's understanding, a historical review of plastic buckling and the associated paradox are outlined in the Appendix.

It is not possible to cover all the topics relevant to our subject in a single book, but we have attempted to provide some examples in Chapter 11 to show further applications of the plastic bending theory. These examples involve circular bars, rings, thin-walled

beams, circular tubes and so forth. Although the configurations of these members are quite different from flat sheets, large plastic deformation and wrinkling remain as the main concern of the investigation.

The overall structure and contents of the book were largely discussed and agreed on by both authors. The Introduction, Chapters 1 - 4, 7 and 8 were written by T.X. Yu; Chapters 5, 6, 9, 10 and the Appendix by L.C. Zhang. Chapter 11 was written collaboratively by both authors. There is an extensive valuable bibliography at the end of each chapter, which we hope will serve as a source of references for further reading.

The contents and style of the book owe much to the influence of many friends and colleagues over the last 15 years. The authors are eternally indebted to Professor W. Johnson, who, from the very beginning, greatly inspired and enthusiastically supervised the first author of the monograph (T.X. Yu) researching this subject. Without his continuous stimulation and encouragement, this book might never have been published. We greatly appreciate the advice and support of our former supervisors, colleagues and friends, who have critically read various sections of the book (or its former versions) and kindly offered us many valuable suggestions; in particular, we wish to record our gratitude to Professor R. Wang (Peking University), Dr. W. J. Stronge (Cambridge University), Professor S.R. Reid (UMIST) and Dr. T.Y. Reddy (UMIST). We also wish to thank Ms. Janet Jones (Sydney University), who read the manuscript of the book and offered valuable comments on its language and style.

Compared with its Chinese version published in 1992 by the Science Press (Beijing), this revised English version includes the most recent developments in the area. We like to thank World Scientific Publishing Co., for their encouragement and patience during our preparation of the manuscript. The authors would be extremely grateful to readers for bringing errors to their notice or indeed making any comments on the contents of the book.

Hong Kong
Sydney, Australia
September 1995

T.X. Yu (Tongxi Yu)
L.C. Zhang (Liangchi Zhang)

This page is intentionally left blank

Contents

Foreword by W. Johnson	vii
Preface	ix
Introduction	1
Chapter 1 Engineering Theory of Elastic-Plastic Bending of Beams	7
§1.1 Early Theories on Bending	7
§1.2 Development of Engineering Theory of Bending	11
§1.3 Pure Bending and Springback of Straight Beams:	
General Formulation	12
§1.4 Symmetric Bending of Elastic, Perfectly Plastic Beams	20
§1.5 Asymmetric Pure Bending and Springback of Beams	34
§1.6 Effect of Strain-Hardening on Bending and Springback	44
§1.7 Influence of Shear Forces	51
§1.8 Influence of Axial Force	55
References	68
Chapter 2 Mathematical Theory of Plastic Bending	72
§2.1 Defects of Engineering Theory of Plastic Bending	72
§2.2 Mechanism of Plastic Bending of Plates	74
§2.3 Mathematical Theory of Plastic Bending of Plates	
under Plane-Strain Condition	76
§2.4 Bending Theory of Strain-Hardening Plates	85
§2.5 Bending Theory of Plates with Real Stress-Strain Relation	88
§2.6 On the Range of Applicability of the Engineering Theory	
of Plastic Bending	100
§2.7 Elastic-Plastic Bending of Beams under Plane-Stress Condition ...	103

§2.8	Slip-line Field and Upper-Bound Solutions to Bending Problems . .	106
	References	116
Chapter 3	Large Elastic-Plastic Deflection of Flexible Beams	119
§3.1	The Necessity of Studying the Large Deflection of Flexible Beams	119
§3.2	Elastica	122
§3.3	Plastica	137
§3.4	Elastic-Plastic Post-Buckling of an Euler Strut	140
§3.5	Large Elastic-Plastic Deflection of a Cantilever Beam Subjected to a Concentrated Force at its Tip	154
	References	165
Chapter 4	Bending of Strips in Cylindrical Dies	168
§4.1	Experimental Study of Bending of Strips in Cylindrical Dies	168
§4.2	Modelling of Bending of Strips in Cylindrical Dies	186
§4.3	Bending of Circular Bars in Cylindrical Dies	208
§4.4	Influence of Strain-Hardening on the Bending Behaviour of Strips	210
	References	220
Chapter 5	Numerical Solutions to Single-Curvature Bending Problems	222
§5.1	The Finite Element Method	222
§5.2	A Semi-Numerical Method	227
	References	240
Chapter 6	Axisymmetric Bending of Circular Plates	242
§6.1	Introduction	242
§6.2	Fundamental Equations of Thin Circular Plates	244
§6.3	Large Elastic-Plastic Deflection of Circular Plates Subjected to Edge Bending Moments	252
§6.4	The Perturbation Method for Circular Plates under Ring Loads . . .	261
§6.5	Iso-deflection Method for Elastic-Plastic Bending	273
	References	278

Chapter 7	Pressing Circular Plates into Hemispherical Dies	282
§7.1	Experimental Study	282
§7.2	Elastic Behaviour of a Circular Plate Loaded by a Hemispherical Punch	289
§7.3	Plastic Deformation and Springback of a Simply-Supported Circular Plate Pressed by a Hemispherical Punch	295
	References	308
Chapter 8	Pressing Rectangular Plates into Doubly-Curved Dies	309
§8.1	Elementary Theory of Plate Bending for Small Deflections	309
§8.2	Stamping Rectangular Plates into Doubly-Curved Dies: Experiments	324
§8.3	Stamping Rectangular Plates into Doubly-Curved Dies: Modelling	346
	References	359
Chapter 9	Numerical Methods for Double-Curvature Bending	360
§9.1	Introduction	360
§9.2	The Dynamic Relaxation Method	361
§9.3	Consistent Technique for Plasticity Computation	374
§9.4	Contact Treatment	377
§9.5	Axisymmetric Stamping of Circular Plates	382
§9.6	Applicability of Deformation Theory of Plasticity	395
§9.7	The Method of Weighted Residuals	403
	References	405
Chapter 10	Wrinkling of Circular Plates and Flanges	411
§10.1	Wrinkling: Causes, Criteria and Problems	411
§10.2	Wrinkling of a Flange during Deep-Drawing	416
§10.3	Wrinkling of Circular Plates under Edge Bending Moments	435
§10.4	Experimental Study of Wrinkling of Circular Plates	438
§10.5	Approximate Analysis of Wrinkling of Circular Plates	451
§10.6	General Solution for Wrinkling of Plates under Arbitrary Loads	454
§10.7	Further Remarks	464
	References	464

Chapter 11 Further Applications of Plastic Bending Theory 468

 §11.1 Cross-Roll Straightening of Circular Bars 468

 §11.2 Large Deformation of a Perfectly Plastic Circular Ring under
 Compression 477

 §11.3 Elastic-Plastic Lateral Buckling of Narrow Beams Subjected
 to Bending and Axial Force 487

 §11.4 Stretch-Bending of Thin-Walled Channels 497

 §11.5 Stamping by Deformable Forming Tools 499

 §11.6 Nosing of Thin-Walled Circular Tubes 505

 §11.7 Flattening and Local Buckling of Pipes in Bending 508

 References 523

Appendix Plastic Buckling of Plates and Shells - An Overview 527

 §A.1 A Brief Overview on the Plastic Buckling of Plates and Shells . . . 527

 §A.2 The Paradox of the Plastic Buckling of Plates and Shells and
 Some Interpretations 531

 References 541

Subject Index 549