Peyresq Lectures on

Nonlinear Phenomena

Volume 2

This page is intentionally left blank

Peyresq Lectures on

Nonlinear Phenomena

Volume 2

Editors

Jacques-Alexandre Sepulchre

Jean-Luc Beaumont

Institut Non Linéaire de Nice, France

World Scientific

New Jersey • London • Singapore • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: Suite 202, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

PEYRESQ LECTURES ON NONLINEAR PHENOMENA (Volume 2)

Copyright © 2003 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 981-238-402-2

ISBN 981-238-416-2 (pbk)

Printed in Singapore by World Scientific Printers (S) Pte Ltd

Preface

This book is the second volume of a compilation of lecture notes on various topics in nonlinear physics delivered by specialists during the summer schools organised by the Institut Non Linéaire de Nice (INLN)^a in Peyresq (French Alps of Provence) since 1998. The first volume, edited by R. Kaiser and J. Montaldi, contains courses from the years 1998 and 1999. This volume collects notes of the lectures given from the summers of 2000, 2001 and 2002. The motivation of this publication is twofold. First, it gives to the participants of the school a written version of the material presented in Peyresq. Second, it increases the benefits of these courses, given originally in French, since they can now reach a broader audience.

At first glance, the diversity of the subjects collected in this single volume might be surprising. This is however a deliberate choice. The main goal of these schools is indeed to promote the communication, and hopefully the transfer of knowledge, among the researchers in nonlinear sciences, in particular among mathematicians, theoreticians, and experimentalists. The field of nonlinear sciences has expanded on numerous fronts. Nevertheless, there should be, and there are, common techniques to analyse nonlinear phenomena; thus, methods designed in one domain can often be transferred to others, or can at least inspire a way of thinking. In this perspective, this book is aimed at giving profound introductions to various domains of the nonlinear physics. It is addressed to any scientist, graduated student, researcher or professor who wishes to acquire the “common culture” of the nonlinear sciences at the professional level.

The present volume contains nine lectures distributed in three main areas: (i) A comprehensive lecture on the dynamo effect is complemented by an introduction to the theory of bifurcations in physical models with symmetries; (ii) Bose Einstein condensates are considered in three lectures, with an extensive description of the experiments, the related technical and theoretical issues, and some insights on the underlying theoretical problems; (iii) Fluid dynamics, which contain a wealth of nonlinear paradigms, are represented by a general discussion on “mixing” (see the cover of this book for a nice experimental illustration) and the timely problem of flows of complex fluids; moreover, a tutorial on the theory of singularity and similarity introduces the reader to the use of powerful concepts in the analysis

^aIn 2001, the Institut de Recherche de Physique Hors Equilibre (IRPHE) of Marseille has become a partner of the School, and in 2002 the Laboratoire de Physique Statistique of the Ecole Normale Supérieure has joined the organising committee.

of physical models. Finally, a new theoretical approach to hydrodynamics based on the concept of permutations is presented.

As a member of the organising committee of the school, I was more specifically involved in the task of gathering and editing these notes. I take this opportunity to thank all the contributors who agreed to write high quality lecture notes.

I also wish to warmly thank Jean-Luc Beaumont, Administrator of the INLN, who spent a great deal of energy in making the organisation and the unfolding of the “Rencontres Non Linéaires de Peyresq” smooth and enjoyable. His expertise as an organiser, and his enthusiasm have greatly contributed to the success of these schools.

Finally, the *Fondation Nicolas-Claude Fabri de Peiresc* is gratefully acknowledged for making available the beautiful village of Peyresq. We also thank the *Formation Permanente du CNRS* for main financial support, and acknowledge a financial contribution from the *Ecole doctorale* of the University of Nice-Sophia Antipolis and from the INLN.

Jacques-Alexandre Sepulchre
Sophia Antipolis, December 2002.

Photo of the Peyresq countryside taken by Denis Tremblay.

“Les Rencontres Non Linéaires de Peyresq”

is a summer school organised by :

L’Institut Non Linéaire de Nice (INLN)

1361 route des Lucioles - Sophia Antipolis, F-06560 Valbonne.

in collaboration with :

L’Institut de Recherche de Physique Hors Equilibre (IRPHE)

Faculté des Sciences de Saint Jérôme, F-13000 Marseille.

Scientific Committee :

Pierre Coullet (INLN)

Paul Clavin (IRPHE)

Organising Committee:

Basile Audoly (LPS-ENS)

Jean-Luc Beaumont (INLN)

Alain Pocheau (IRPHE)

Jacques-Alexandre Sepulchre (INLN)

This page is intentionally left blank

Contents

Preface	v
1 The Dynamo Effect	1
<i>S. FAUVE and F. PETRELIS</i>	
1 Introduction: from industrial dynamos to the magnetic field of stars and planets	1
1.1 Generation of an electric current by a rotating conductor	1
1.2 Magnetic fields of astrophysical objects	4
1.3 Fluid dynamos	6
2 The MHD approximation	8
2.1 The approximations and equations of magnetohydrodynamics	8
2.2 Relation with a two-fluid model	11
2.3 Boundary conditions	12
2.4 Relevant dimensionless numbers	14
2.5 Some limits of the MHD equations	16
3 Advection and diffusion of a passive vector	17
3.1 More or less useful analogies	18
3.2 Accelerated diffusion	19
3.3 Local amplification	20
3.4 Expulsion of a transverse magnetic field from a rotating eddy	22
3.5 Effect of differential rotation and the Herzenberg dynamo	24

4	Necessary conditions for dynamo action	27
4.1	Magnetic energy	27
4.2	The critical magnetic Reynolds number	29
4.3	“Antidynamo theorems”	30
5	Laminar dynamos	31
5.1	The Ponomarenko dynamo	32
5.2	Spatially periodic dynamos	38
6	Turbulent dynamos	46
6.1	Dynamos generated by small scale turbulence?	46
6.2	The dynamo threshold at large kinetic Reynolds number	47
6.3	Effect of turbulence on the dynamo threshold	48
6.4	Is it useful to have scale separation?	50
7	Saturation of the magnetic field above the dynamo threshold	51
7.1	Laminar dynamos	52
7.2	Saturation in the high Re or small P_m limit	59
8	Conclusion	61
2	An Introduction to Equivariant Bifurcations and Spontaneous Symmetry Breaking	67
	<i>P. CHOSSAT</i>	
1	Introduction	67
2	The classical bifurcation theory	68
2.1	Elementary bifurcations	68
2.2	Bifurcation with symmetry: a general approach	74
2.3	Existence theory	79
3	Equivariant dynamical systems	83
3.1	The Hopf bifurcation	84
3.2	Relative equilibria and relative periodic orbits	90

3 Bose-Einstein Condensation of Trapped Atomic Gases 97

Ph. W. COURTEILLE

1	Introduction	97
2	Creating Bose-Einstein condensates	99
2.1	Bose-Einstein condensation in alkalis	100
2.2	Other systems	102
3	Measuring Bose-Einstein condensates	104
3.1	Density and velocity distributions	104
3.2	Imaging techniques	108
3.3	Measuring other quantities	110
4	Manipulating Bose-Einstein condensates	111
4.1	Dipole force	111
4.2	Coherent coupling	112
4.3	Resonant scattering	116
4.4	Manipulation of collisions	118
5	Experiments on Bose-Einstein condensates	121
5.1	Thermodynamics	121
5.2	Dynamics and superfluidity	123
5.3	Atom optics	124
5.4	Interactions with light	130
5.5	Collision resonances	132
6	Conclusion	137

4 An Introduction to Bose-Einstein Condensation Experiments 143

D. GUÉRY-ODELIN

1	Introduction	143
2	Magnetic trap	144
2.1	Preliminary	144
2.2	The Ioffe trap	145
2.3	Noise-induced heating	146
2.4	Collisions	147

3	Adiabatic compression	147
3.1	Preliminaries	147
3.2	How to modify the phase space density by adiabatic compression?	148
3.3	Collision rate	149
4	Collective oscillations	150
4.1	Averaging method	151
4.2	Monopole mode	152
4.3	Dipole mode	152
4.4	Quadrupole mode	152
4.5	The role of the classical mean field	154
5	Evaporative cooling	154
5.1	Evaporation in the collisionless regime	154
5.2	Role of collisions, forced evaporative cooling	156
6	Bose-Einstein condensation	157
6.1	The Gross-Pitaevskii equation	157
6.2	Hydrodynamic description	157
6.3	Superfluidity	157
7	Collective excitations: BEC and the classical gas in the hydrodynamic regime	158
7.1	Introduction	158
7.2	Equation of continuity	159
7.3	Equation for the velocity field	159
7.4	Collective modes	161
8	Rotational properties of a BEC	162
8.1	Moment of inertia	162
8.2	Introduction to vortices	163
8.3	The rotating bucket experiment	163
8.4	Vortex nucleation	164
5	Highlights of the Theory of Bose-Einstein Condensates	169

Y. POMEAU

1	Introduction and outline of the set of lectures	169
---	---	-----

2	Equilibrium thermodynamics of a dilute B–E gas at low temperature	171
2.1	Thermodynamics of the dilute Bose gas with an energy perturbed to first order	172
2.2	Thermodynamics of the dilute Bose gas	175
3	Kinetic theory of a dilute Bose gas	181
3.1	Boltzmann-Nordheim kinetic theory and B–E condensation	182
3.2	Dynamics before collapse	184
3.3	Post collapse dynamics	187
4	Kinetic theory and the Gross-Pitaevskii equation	189
4.1	The kinetic theory at the mean field order	190
4.2	Hydrodynamics of the coupled kinetic equations	196
5	The Gross-Pitaevskii equation and superfluid mechanics: an example	201
5.1	The general solution of the Euler-Tricomi equation near a body	209
6	Summary and conclusion	211
6	Mixing	213

E. VILLERMAUX

1	Interaction leads to uniformity	213
2	The elementary interaction	214
2.1	Composition rule	222
3	Practical mixtures: limit cases	224
3.1	The confined mixture	225
3.2	The ever dispersing mixture	230
3.3	Kinetics	233
3.4	Spectral consequences	236
4	Changing dimensions	238
5	Kinetic equation	242
6	Conclusion	244

7 Flow of Complex Fluids 251*J. MEUNIER and D. BONN*

1	Introduction	251
1.1	Brief review of Newtonian fluid flow	251
1.2	Interaction structure/flow: criteria for non-Newtonian effects	253
1.3	Rheological measurements	255
1.4	Elongational viscosity	257
2	Examples of interaction flow/structure	258
3	Non-Newtonian shear viscosity	263
4	Elastic effects in non-Newtonian fluids	265
4.1	Viscoelastic fluids: first normal stress difference	265
4.2	Normal stress effects on free boundary problems	268
4.3	Elongational viscosity	273
5	Yield stress fluids	277

8 Elements of Similarity and Singularity 285*A. VERGA*

1	Introduction	285
1.1	Thales and the origin of similarity	285
1.2	The selection of relevant parameters	287
1.3	Similarity solution of the diffusion equation	288
2	Free surface problems in the lubrication approximation	291
2.1	Thin film equations	291
2.2	Nonlinear diffusion and the spreading of a drop	295
2.3	The capillary regime of drop spreading	299
3	Second type similarity in a porous medium	303
3.1	The renormalization group	303
3.2	The Barenblatt equation	305

3.3	Perturbation expansion and renormalization group solution	307
4	Shocks, Burgers equation and finite time singularities	310

9 Topics on Volume Preserving Maps and Permutations Related to Hydrodynamics and Electrostatics 319

Y. BRENIER

1	A dynamical system involving permutations	319
2	Volume preserving maps	320
2.1	Definition	320
2.2	Volume and orientation preserving diffeomorphisms	321
2.3	Permutation maps	322
2.4	Dense subsets of volume preserving maps . . .	322
2.5	Doubly stochastic measures	323
3	Geometric interpretation of the Euler equations . . .	325
3.1	Usual description of the Euler equations . . .	325
3.2	Geometrical interpretation of the Euler equations	326
3.3	The Shortest Path Problem	329
4	Approximate geodesics and nonlinear electrostatics . .	330
4.1	Definition of approximate geodesics	330
4.2	The time-discrete SPP	331
4.3	The closest point problem and the polar factorization of Maps	332
4.4	A nonlinear cutoff for electrostatics	333
4.5	The discrete approximate geodesic equation . .	334