DISLOCATION BASED FRACTURE MECHANICS

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To:

**my wife
our children
our grandchildren**

**my parents
my brother**

**Julia
Julia and Bruce
Johannes and Willem
and their mother Leslie
Roelof and Christina
Willem and his wife Eleanor**

The Magic Lengths of Fracture Mechanics

$$\frac{K^2}{\pi \sigma_A^2} \rightarrow \text{crack half length}$$

$$\frac{K^2}{\sigma_y^2} \rightarrow \approx \text{plastic zone size}$$

$$\frac{K^2}{G \sigma_y} \rightarrow \approx \text{crack opening displacement where plastic zone boundary crosses crack plane behind tip}$$

$$\frac{K^2}{G^2} \rightarrow \approx \text{crack tip radius after blunting by dislocation emission}$$

Mnemonics

$\mathcal{G}$ (energy release rate): George Irwin & A. A. Griffith

J Integral: Jock Eshelby & James Rice

K (stress intensity factor): George Irwin & Joseph Kies

PREFACE

A crack in an elastic solid can be considered to be and, indeed, actually is a distribution of dislocations on the crack plane. A crack in an elastic-plastic solid not only is a distribution of dislocations on the crack plane but, in addition, is a distribution of non-redundant dislocations within the plastic zones of the crack tips. The stress-strain-rotation field of a discrete dislocation of unit strength is the Green's function from which the stress and elastic strain-rotation fields of a crack can be obtained. Moreover, the plastic strain-rotation field can be found from the motion and density of the dislocations in the crack tip plastic zones. To ignore the dislocation origin of the large stresses near a crack tip when attempting to find a crack solution is like attempting to solve a solid state physics problem oblivious to the fact that electrical fields arise from electrical charges.

Because the dislocation is the basic building block of the crack, fracture mechanics logically can be developed from its dislocation base. In dislocation texts it is noted that cracks in elastic solids can be modeled with dislocations[†]. Texts on fracture mechanics, however, largely ignore the dislocation foundation of the crack. (Lardner's book is an exception. Lardner's book was written well before the current interest and recent research carried out on dislocation crack tip shielding and before any solution was found for a crack in an elastic-plastic solid using dislocation density fields.) In no dislocation text or fracture text is it noted that the problem of a crack in an elastic-plastic solid can be solved with use of dislocations. This text is the only one to present solutions for cracks in elastic-plastic solids that are based on dislocation density fields. The dislocation density fields are obtained with the aid of the fundamental 'Maxwell's equations for dislocations' presented in Chapter 2. (These 'Maxwell dislocation equations' allow for the existence of a curiosity: a 'point' dislocation complex, or, conversely, the existence of an electron that is dislocation complex.)

I believe that any student entering the field of fracture will benefit from a text in which fracture mechanics is developed from its dislocation base. This book is my effort to supply such a text for the new graduate student or the investigator who is just entering the field. Portions of this book were used in advanced fracture courses offered at Northwestern University. I hope that the student studying this book can develop a dislocation 'world view' of cracks from which to see solutions of crack problems. The chapter on dislocations should give the student more than adequate background for understanding the application of dislocations to crack problems.

When I started to write this text only the problem of the stationary mode III crack in an elastic-plastic solid had been solved completely (without use of

[†] See the texts of Friedel (1964), Hirth and Lothe (1982) and Nabarro (1967). Apparently, the first dislocation based solution for the problem of the crack in an elastic solid is that obtained by Leibfried. His solution employs a planar distribution of infinitesimal dislocations and the Hilbert transforms (Leibfried 1951).

dislocations). No dislocation-based solution existed for any type of crack in an elastic-plastic solid. I put off completion of this book for some years until I found dislocation-based solutions for the three crack types in various elastic-plastic solids[‡]. That is, for the crack opened in tension (mode I crack), the crack sheared in the direction perpendicular to the crack front (mode II crack), and the crack sheared in the direction parallel to the crack front (mode III crack). These dislocation-based solutions are the subjects of three chapters. It is evident from these solutions that complete, exact solutions for mode I and mode II cracks in general elastic-plastic solids are not likely to be found without explicit introduction of dislocations into the solution.

Any crack problem in an elastic solid can be solved starting from the stress field of a discrete dislocation. The stress field of dislocations distributed on a plane leads directly to the Hilbert transforms[†] and to the Muskhelishvili equations that are the tools to obtain crack solutions in elastic solids. A chapter in the book develops in detail how the Hilbert transforms and the Muskhelishvili equations are derived for the student who wants his understanding developed ab initio. Only the results of this chapter, not its derivations, are needed in the other chapters. The solution of the Bilby-Cottrell-Swinden-Dugdale crack, which is the simplest dislocation-based model crack with a (one dimensional) plastic zone, is easily obtained from the Muskhelishvili equations. Moreover, with these equations, as shown in Chapter 5, the amount of dislocation crack tip shielding or antishielding is found in a straightforward way. At this point in the book all the dislocation tools are in place to find solutions for cracks in elastic-plastic solids. The following chapters present all the published and unpublished dislocation-generated solutions for cracks in elastic-plastic solids. These solutions are of infinitely long cracks in an isotropic solid.

The last section of the last chapter of the book discusses one paradox whose existence is only revealed in a dislocation approach to fracture. This puzzle is: the non-redundant dislocations are more numerous than redundant dislocations close to a crack tip if elastic-plastic stress-strain curves obtained under uniform strain conditions are valid under non-uniform strain conditions. But the redundant dislocation density is expected to be always greater than the non-redundant dislocation density. For experimentalists this discrepancy implies that they should provide theorists not only stress-strain curves obtained on uniformly deformed specimens but they should also give them information how stress-strain curves

[‡] Exact analytical solutions were found for all three types of stationary cracks in a linear elastic-plastic solid and for the mode III crack in a work hardening solid and in an elastic perfectly plastic solid. Approximate analytical solutions were obtained for the growing mode III crack and for the stationary mode I crack and for the mode II crack in an elastic perfectly plastic solid.

[†] Appendix A at the end of the book is a short table of Hilbert transforms. Appendix B is a long table of integrals that arise in crack problems but are not found in the usual integral tables.

change if the plastic deformation is spatially very nonuniform. For theorists this paradox can be the starting point for predicting critical stress intensity factors from experimental stress-strain curve data. Until the redundant and non-redundant dislocation paradox is resolved there is little point in attempting to obtain corrections to any near crack tip stress-strain field solution that take into account the fact that strains are large in this region. Any proposed large strain constitutive equation used near a crack tip must lead to the result that the non-redundant dislocation density is greater than the redundant dislocation density. (It should be noted that in this book the derivations of dislocation density fields from plastic strain contain the implicit assumption that strain is small. It is shown in the last section of the book (§ 10.6 of Chapter 10) that the region around a crack tip in which the strain is large is very small compared with the plastic zone size.)

My own research in the field of fatigue cracks and fracture, upon which part of this text is based, was supported in years past by the National Science Foundation, the Office of Naval Research, the Department of Energy, and the Air Force Office of Scientific Research. I thank these agencies for their former support. I thank Professor James S. Koehler, the co-proposer of the Zener-Stroh-Koehler crack and my thesis advisor, for introducing me to the wonderful world of the dislocation. I first came across elastic-plastic crack problems through the speakers at the mechanics colloquia that Dr. George Irwin, the father of fracture mechanics, ran at the U. S. Naval Research Laboratory. I confess that I understood virtually nothing of what was discussed at those long ago seminars. I was introduced, also many years ago, to the Muskhelishvili analysis used in part of this book while attempting to solve earthquake fault slippage problems during a sabbatical leave at the California Institute of Technology. I am grateful to Professor Robert Sharp, then department chairman of the Geological Sciences Division, for the invitation to spend a period of time at Caltech. That short period was the starting point for all my dislocation-based theoretical fracture work.

Over the years, I have been very fortunate for my own research to have the benefit of the stimulation from my, and my wife's, colleagues, students, coauthors at Northwestern University, Los Alamos National Laboratory, U. S. Army Cold Regions and Research Laboratory (CRREL), Scott Polar Research Institute, U. S. Naval Research Laboratory (NRL) and Paul Scherrer Institute. I am particularly indebted to Morris Fine and to Jerome Cohen for my career at Northwestern University and to Oscar Marzke for my career at NRL; to Walter Green for many research summers at Los Alamos; and to Anthony Gow, Chester Langway, Hans Röthlisberger, Wilford Weeks for my glaciological education at CRREL (and its predecessor: the U. S. Army Snow Ice and Permafrost Research Establishment (SIPRE)). I thank Jan Achenbach, Norman R. Adsit, Sean Agnew, Carolyn R. Aita, George S. Ansell, Anthony C. Arko, Richard J. Arsenault, Fred Boehme, John Barker, Zdenek Bazant, James Bender, H. M. Berg, G. Edward Birchfield, John Blacic, John E. Bloom, Mitchell L. Brandt, John E. Breen, John Brittain, Neil R. Brown, Robert Brown, Larry H. Burck, Jose Cabanas, Kent R. Carson, Hector

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Evanston, Illinois
November 1995

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SELECTED SYMBOLS LIST

a	crack length, one half of total length of internal crack (crack tips at $x = \pm a$) or total length of surface (edge) crack
a_R, a_L	$x = a_R, x = -a_L$ are positions of right and left hand crack tips when origin centered on plastic zone edges
da/dN	growth rate per cycle of a fatigue crack
b	Burgers vector of a discrete dislocation
b	length of Burgers vector of a discrete dislocation
b_θ	azimuthal component of Burgers vector
b_r	radial component of Burgers vector
b_T	total Burgers vector of dislocations in a Zener-Stroh-Koehler crack
$B(x)$	dislocation density of infinitesimal dislocations on a plane (in total Burgers vector per unit length);
$\mathcal{B}$	dislocation density of infinitesimal dislocations
c	outer edge of plastic zones of internal crack are at $x = \pm c$
C_L	longitudinal wave velocity
C_R	Rayleigh wave velocity
C_S	shear wave velocity
CTOD	crack tip opening displacement (at $x = a$)
$D(x)$	displacement across the crack plane. Equal to integral of $B(x)$.
F, T	subscripts for terms involved with finger and thumb trajectories
F_E	dislocation crack extension force
F_D	dislocation crack deflection force
g	gravitational acceleration
G	shear modulus (elastic)
G^*	shear modulus (linear plastic)
$\bar{G}$	shear modulus (linear elastic-plastic)

$\bar{G}$	effective shear modulus $\bar{G} = 2G_A G_B / (G_A + G_B)$, interface crack
$\mathcal{G}$	Irwin's energy release rate
I, II, III	subscripts for terms involved with modes I, II and III cracks
$\mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L}$	integrals for elastic displacements of Griffith-Inglis and Zener-Stroh-Koehler cracks
J	'J' from J Integral
K	nominal stress intensity factor: For crack in infinite solid, $K = \sigma_A (\pi a)$
K_{gc}	critical stress intensity factor for Griffith-Inglis crack for cleavage crack advance
K_c	stress intensity factor at fracture
K_{gb}	critical stress intensity factor for Griffith-Inglis crack for blunting
K_t	local stress intensity factor at crack tip
ΔK	cyclic stress intensity factor: $\Delta K = (\sigma_{\max} - \sigma_{\min}) \sqrt{(\pi a)}$
ΔK_{th}	threshold cyclic stress intensity factor below which a long fatigue crack will not grow
L	dislocation (or impurity atom) shielding or antishielding stress intensity factor
n	unit normal to contour
PZS	plastic zone size
$\mathcal{R}_F, \mathcal{R}_T$	radius of curvature of finger and thumb trajectories
t	unit tangent vector
u	displacement vector (components u, v and w in the x, y and z directions or u_r, u_θ and w in the r, θ and z directions)
V	dislocation velocity or crack velocity
α	$\alpha = (1 - \nu)$ where ν is Poisson's ratio
α_e	$\alpha_e = \alpha = (1 - \nu)$
α_i	$\alpha_i = \alpha_e$ or α_s
α_s	$\alpha_s = 1$

$\tilde{\alpha}$	$\tilde{\alpha} = \frac{1}{2}\alpha = \frac{1}{2}(1-\nu)$ for mode II, crack $\tilde{\alpha} = 1$ for mode III crack.
$\bar{\alpha}$	$\bar{\alpha} = (1+\nu)$ in plane stress problems
$\bar{\alpha}$	$\bar{\alpha}$ = effective alpha constant in interface crack problems
β	Dundurs parameter, $\beta = \frac{1}{2}[(1-2\nu_A)G_B - (1-2\nu_B)G_A]/[\alpha_A G_B + \alpha_B G_A]$
β_L	$\beta_L = (1-V^2/C_L^2)^{1/2}$
β_S	$\beta_S = (1-V^2/C_S^2)^{1/2}$
β_{2S}	$\beta_{2S} = (1-V^2/2C_S^2)^{1/2}$
γ	surface energy
ϵ_{ij}	tensor strain components
$\epsilon, \epsilon^e, \epsilon^p$	strain magnitude, elastic strain, plastic strain
λ, μ	Lamé elastic constants
ν	Poisson's ratio
ρ_i	density of either solid, liquid, ice, water, etc.
ρ_N	dislocation density of discrete non-redundant dislocations in plastic zone (in total number per unit area)
ρ_R	dislocation density of discrete redundant dislocations in plastic zone (in total number per unit area)
σ_{ij}	tensor stress components
$\sigma_{i\omega}$	$\sigma_{i\omega} \equiv (G/\alpha)\{\omega_i\}_{\text{elastic}} = \text{rotation pseudo-stress component}$
σ	stress magnitude
σ_A	applied stress
$\omega, \omega^e, \omega^p,$	rotation, elastic rotation, plastic rotation
ω_{elastic}	elastic rotation
ω_{plastic}	plastic rotation