CHAOTIC DYNAMICS IN HAMILTONIAN SYSTEMS

With Applications to Celestial Mechanics

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CHAOTIC DYNAMICS IN HAMILTONIAN SYSTEMS

With Applications to Celestial Mechanics

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to all the shoemakers in my family

Preface

*Science may set limits to knowledge,
but should not set limits to imagination.*

Bertrand Russell

The story in this book is a product of the imagination of many minds. People have been occupied with the apparent changing nature of their environment ever since the dawn of humanity. In some instances this occupation took its refuge in theological explanations and metaphysical philosophy. The dynamical aspects of existence were questioned and polemicized from many a perspective, from the skeptics' viewpoint of motion as a deception of the senses, to Heraclitus' claim on change as the fabric of existence. The development of an empirically based scientific method eventually allowed relevant observations to be formalized and the language that arose, mechanics, to be used to further our understanding of motion.

The nature of dynamics, while definitely not understood in a metaphysical sense, has nevertheless yielded to some of science's most ingenious imaginations. The laws of mechanics, as described in the language of mathematics, and in particular, differential equations, have successfully been applied to very large and distant objects, as well as those small and nearby. So successful has this endeavor been that sometimes there has arisen a precarious belief in the "truth" contained in these laws. The laws, in all their simplicity, have been relegated to an ideal world of platonic ideas where they transcend their incomplete incarnations in the present world. Or, in the words of Bertrand Russell in his *History of Western Philosophy*:

Most sciences, at their inception, have been connected with some form of false belief, which gave them a fictitious value. Astronomy was connected with astrology, chemistry with alchemy. Mathematics was associated with a more refined type of error. Mathematical knowledge appeared to be certain, exact, and applicable to the real world; moreover it was obtained by mere thinking, without the need of observation. Consequently, it was thought to supply an ideal, from which every-day empirical knowledge fell short. It was supposed, on the basis of mathematics, that thought is superior to sense, intuition to observation. If the world of sense does not fit mathematics, so much the worse for the world of sense.

But mechanics is part of the world of sense. Its everpresence in our daily lives makes it an urgent matter to which science in its many guises inevitably finds itself drawn. Dynamical phenomena range from the apparently simple and predictable motion of a vibrating string in a musical instrument to the complex flow of fluid through the heart of an animal. Despite the enormous scope of mechanics, be it the celestial mechanics of stellar bodies or the turbulent flow of air in human lungs, the predictive powers of the mathematical laws of mechanics have completely justified their continued use.

The realization that complex behavior is contained within a seemingly simple mathematical formulation without the need to introduce random effects due to external disturbances has dawned on its investigators primarily in the past hundred years. Human imagination, largely that of Henri Poincaré, has conceived of intuitive tools to describe the origin of such complex dynamics. A fascinating world of possibilities has appeared, a pandorian box of secrets previously unknown. Where science has still only begun to nudge the boundaries of this field, the potential applications are as numerous as they are encouraging.

This book attempts to describe some aspects of the theoretical foundations of the phenomena described above. In the spirit of the previous remarks, the theory attains its remarkable beauty primarily in its interactions with real physical examples. A rich and complicated weave of mathematical ideas truly sparkles only in the light of the stars. This is also the case in this text, where the complex dynamics of small circumasteroidal grains under the perturbation of radiation pressure originating in the sun are studied. Understanding its features is of astronomical interest as well as practical importance, should missions ever to try to approach asteroids at close distances.

Communication is difficult enough between people who share a common language. Imagine then the endless confusion possible in its absence. The language of this book is primarily that of mathematics and, in particular, the theory of dynamical systems. There is probably no replacement for reading it from the beginning if only occasionally skimming where familiar facts are discussed. I have attempted to retain conventional notation and terminology throughout the text, but what is conventional for one may be atypical for another.

This book is not, however, a book of mathematics. It is an attempt to describe a set of complex ideas in enough detail to satisfy the more inquiring minds, for whom ambiguity through a popular treatment falls short. Consequently, many results are not proved from fundamental definitions, but rather further references are made to the existing literature. Nevertheless, it is my hope that a suitable balance has been achieved between the rigor of a mathematical treatment and the phenomenology of intuition.

The intended audience is a first or second year graduate student with a basic background in differential equations and classical mechanics. I believe there is much to

interest the applied mathematician as well as the theoretical mechanical engineer and astronomer. Appropriate judgment must be used in determining the best approach to reading this book, depending on one's background and interest.

In the first chapter of this book the leitmotif of each character of the symphony is introduced. The theory is not meant to be complete; instead, only relevant properties are considered and preliminary analysis, as pertains to later chapters, is presented. This chapter covers the geometrical theory of dynamical systems, some basic facts of differential equations, the laws of mechanics and particularly the Hamiltonian formulation, and finally the application to celestial mechanics.

Chapter 2 picks up the tune of Hamiltonian mechanics and describes in detail some techniques associated with its analysis. In particular, the first dissonant chords in the previously regular melody are introduced in connection with the KAM theorem. In particular, the remnants of predictable behavior contained in the KAM result are discussed using methods developed in the previous decade that are alternative to those used in its original proof.

In Chapter 3 we widen our perspective to general dynamical systems and particularly to those satisfying a certain property, that of exponential splittings. The chapter aims at proving the existence of chaotic and highly unpredictable behavior in a simple mathematical system given certain geometric facts. As in the previous chapter, an alternative method of proof to the horseshoe construction usually given in the literature, is provided. Particularly useful are the geometric ingredients in the proof that return again in later chapters. Finally, the numerical tool of Liapunov exponents is introduced as a convenient means of detecting chaos in physical applications.

A perturbation method applicable to proving the presence of chaos in a dynamical system is considered in Chapter 4. The Melnikov method, first conceived in the 1960s and fully developed in the past decade, is presented in detail, with the additional aim of laying the groundwork for chapters to come. The theory is exemplified in a particular application requiring the alternative treatment to standard derivations given in this chapter.

In the fifth chapter the astronomical problem alluded to above is introduced. The physical background is outlined and the methods of the previous chapters are applied to the particularly simple case of planar motion. We argue for the presence of irregular motion in the dynamics of small grains orbiting asteroids and perturbed by radiation pressure. While the consequences are relatively mild in the two-degrees-of-freedom case, similar results in Chapter 7 are found to be of possible physical importance.

In fact, Chapter 6 generalizes the methods and ideas of Chapters 4 and 5 to the problem of more degrees-of-freedom, in particular three or higher. The higher-dimensionality of the relevant systems of ordinary differential equations dramatically increases the possibility of irregular behavior, particularly in the form of large excursions in the classical adiabatic invariants. These results are finally applied to the full three-dimensional motion of the circumasteroidal grains in Chapter 7. More specif-

ically, the possibility of grains escaping the vicinity of the asteroid due to radiation pressure is investigated and the relevant time scales are estimated.

This book concludes with a brief outline of some open-ended questions that are at the forefront of research in the areas described in the previous chapters. It is suggested that the reader continually refer to this last section, so as to retain an appropriate sense of incompleteness, rather than, as often is the case in books of mathematics, to be deceived by the complexity of the theory to believe in its completeness.

Finally, a word on the textual layout employed. In particular, I have chosen to intersperse *illustrations* into the running text. These are **not** simply examples of the theory. Instead, they often contain results used in the following text and should thus be read along with the rest of the text.

I find existence fascinating and awesome. It is hard if not impossible to fathom the delicate yet immensely powerful workings in all aspects of this giant process. Equally unfathomable is the apparent structure and susceptibility to the theories of science which nature displays. In all modesty I have written this book to share some remarkable castles in the air with which some of us are preoccupied and which we believe to be useful in describing nature.

There are many books that treat the ideas described here in more or less detail. I have attempted to write a book I would have liked to read, but in the course of its writing my perspective has changed and I am no longer who I was at its conception. It is hard for me to judge the success of my endeavor. Nevertheless, there it is.

Many people have been very helpful in teaching me the ideas contained herein. In particular, Martin Lesser, who awoke my interest in the field of dynamical systems, has continued to encourage me throughout and moreso with respect to the writing of this book. For this, and many other things, I owe him much gratitude. Philip Holmes, my doctoral advisor, guided me and stimulated my quest in dynamical systems. An inspiring man with many-faceted interests in science and an ability to convey this fascination to others, his advice and example deserves my appreciation. Further thanks go to Richard Rand, George Haller, Alessandro Morbidelli, Pierre Lochak, Hanno Essén, Håkan Eliasson, Martin Zimmerman, and many others for valuable discussions and their invaluable knowledge. Many thanks also to my mother-in-law whose technical editing greatly enhanced the readability of this text.

I believe that my writing this book has been fairly inconsequential to most around me and has not perturbed their lives greatly. Everyone, that is, except our parakeet, Jamie, who has had to stay in his cage more often than we would have liked.

Finally, to my wife and mother who are closest to me in my life, for their unconditional support. I love them deeply.

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