DIAMOND

A Paradox Logic

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A Paradox Logic

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Introduction

There once was a poet from Crete
who performed a remarkable feat
He announced to the wise
"Every Cretan tells lies"
thus ensuring their logic's defeat.

"It cannot be too strongly emphasized that the logical paradoxes are not idle or foolish tricks. They were not included in this volume to make the reader laugh, unless it be at the limitations of logic. The paradoxes are like the fables of La Fontaine which were dressed up to look like innocent stories about fox and grapes, pebbles and frogs. For just as all ethical and moral concepts were skillfully woven into their fabric, so all of logic and mathematics, of philosophy and speculative thought, is interwoven with the fate of these little jokes."

- Kasner and Newman, "Paradox Lost and Paradox Regained"
from volume 3, "The World of Mathematics"

This book is about "diamond", a logic of paradox. In diamond, a statement can be true yet false; an "imaginary" state, midway between being and non-being. Diamond's imaginary values solve many logical paradoxes unsolvable in two-valued boolean logic.

The purpose of this book is not to bury Paradox but to praise it. I do not intend to explain these absurdities away; instead I want them to blossom to their full mad glory.

I gather these riddles together here to see what they have in common. Maybe they'll reveal some underlying unity, perhaps even a kind of fusion energy! They display many common themes; reverse logic, self-reference, diagonality, nonlinearity, chaos, system failure, tactics versus strategy, and transcendence of former reference frames. Although these paradoxes are truly insoluble as posed, they do in general allow this (fittingly paradoxical!) resolution; namely through loss of resolution! To demand precision is to demand partial vision. These paradoxes define, so to speak, sharp vagueness.

A sense of humor is the best guide to these wild regions. The alternative seems to be a kind of grim defensiveness. There exists a strange tendency for scholars to denigrate these paradoxes by giving them derogatory names. Paradoxes have been dubbed "absurd" and "imaginary" and even (O horror!) "irrational". Worse than such bitter insults are the hideously morbid stories which the guardians of rationality tell about these agents of Chaos. All too many innocuous riddles have been associated with frightening fables of imprisonment and death; quite gratuitously, I think. It is as if the discoverers of these little jokes hated them and wanted them dead. Did these jests offend some pedant's pride?

Paradox is free. It overthrows the tyranny of logic and thus undermines the logic of tyranny. This book's paradoxes are more subversive than spies, more explosive than bombs, more dangerous than armies, and more trouble than even the President of the United States. They are the weak points in the status quo; they threaten the security of the State. These paradoxes are why the pen is mightier than the sword; a fact which is itself a paradox.

This book has two sections: elementary and advanced. The "elementary diamond logic" section covers the classic paradoxes of mathematical logic, defines diamond's values and operators, notes diamond's equational laws, introduces diamond's "phase order" lattice, proves that diamond resolves all self-referential systems, resolves the classic paradoxes, and demonstrates that diamond embeds the continuum. This section ends with "Zeno's theorem", which finds diamond-valued fixedpoints for any real-valued function. The "advanced diamond logic" section covers diamond's boolean underpinnings, describes "diffraction" and other non-boolean forms of diamond reasoning, embeds 3-valued logic within diamond, and relates diamond to Gödelian metamathematics, non-zero-sum games, and other topics.

Thus this book illustrates a classic paradox of development; namely, the elementary is advanced, and the advanced is elementary. The results of Part I are basic, but I have known them long enough to refine and strengthen the proofs; thus the elementary is advanced. The results of Part II are subsequent research, and are therefore less developed; that is, less certain and less general. Thus the advanced is elementary.

I would be a liar indeed not to acknowledge my many friends and colleagues. These include Douglas Hofstadter, Louis Kauffman, Tarik Peterson, Sylvia Rippel, Rudy Rucker, Dick Shoup, Raymond Smullyan, Stan Tenen, and Francisco Varela; without their vital input over many years, this book would have been impossible.

Love and thanks go to my parents, Earl and Marjorie, without whom I would have been impossible.

Special thanks go to my fiancée Sherri Krynski, without whom I would not have published this.

Finally, due credit (and blame!) go to myself, for boldly rushing in where logicians fear to tread.

Said a monk to a man named Joshu
"Is that dog really God?" He said "Mu."
This answer is vexing
And highly perplexing
And that was the best he could do.