THE MYSTERY OF KNOTS

Computer Programming for Knot Tabulation

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THE MYSTERY OF KNOTS

Computer Programming for Knot Tabulation

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Charilaos Aneziris

P R E F A C E

This manuscript is mainly addressed to young mathematicians, mostly late undergraduates or early graduates, who are interested in specializing and pursuing a career in Knot Theory. It is also relevant to current Knot Theorists interested in the applications of Computer Science to their field, to Computer Scientists eager to be introduced to one of the most fascinating mathematical problems where their programming skills can be applied, and to scientists in other disciplines interested in a general overview of Knot Theory.

Our main focus is the discussion of the problem of tabulation of knots, and to demonstrate how through developing appropriate algorithms to be run as computer programs, one may obtain concrete results. The mathematical background needed to understand the discussion is quite limited. Advanced mathematical knowledge from Computer Programming, Topology, Group Theory and other areas, is presented as simply as possible, just before it is being used. The only knowledge expected from the reader, is a very elementary understanding of Euclidean Geometry, an intuitive feeling of the idea of continuity, and most important a good understanding of Mathematical Logic and Reasoning.

The material is organized as follows. Following an introduction that includes a historical overview and the statement of the main problem, there is a very extensive presentation of the relevant aspects of Knot Theory in the first part. This presentation is not a synopsis of Knot Theory, since we omit important aspects that are not relevant to the discussion. Ideally, after the end of the first part, one is equipped with all the Knot Theory knowledge that is needed to tackle the problem of knot tabulation.

The second part is exclusively concentrated to the development of the

appropriate algorithm, and especially to how the numerous obstacles and hindrances are overcome. Towards the end, the reader may find what in computer slang is called a “pseudocode”, followed by a brief summary of the results obtained after we ran the computer program. Finally, there is a third part that contains an extensive table of the tabulated knots, including a proof that they all differ from each other.

Since Knot Theory takes place in the familiar three-dimensional space, we present a number of figures, whose purpose is to encourage the reader to form a visual picture of the concepts involved. It is our belief that such a visualization leads to a better understanding than mere statements of formal definitions and theorems. In addition, we provide a number of exercises to motivate the reader to be an active participant in the discussion. While some of these exercises are quite involved, and it is not necessary to solve them in order to understand the subsequent theory, readers are encouraged to spend time attempting them, and reading and understanding the solutions provided.

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